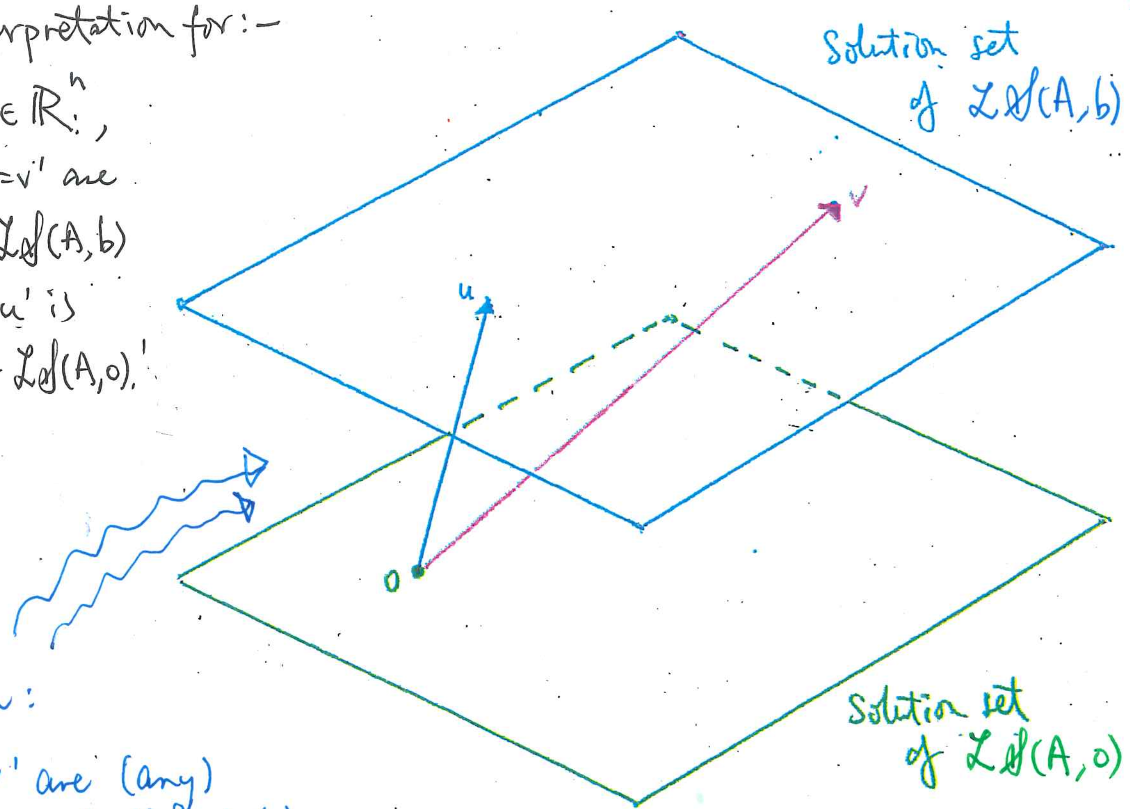
Given : A is an $(m \times n)$-matrix, and b is a vector in $\mathbb{R}^m$.

Geometric interpretation for :–

'For any $u, v \in \mathbb{R}^n$, if '$x=u$', '$x=v$' are solutions of $\mathcal{LS}(A,b)$ then '$x=v-u$' is a solution of $\mathcal{LS}(A,0)$.'

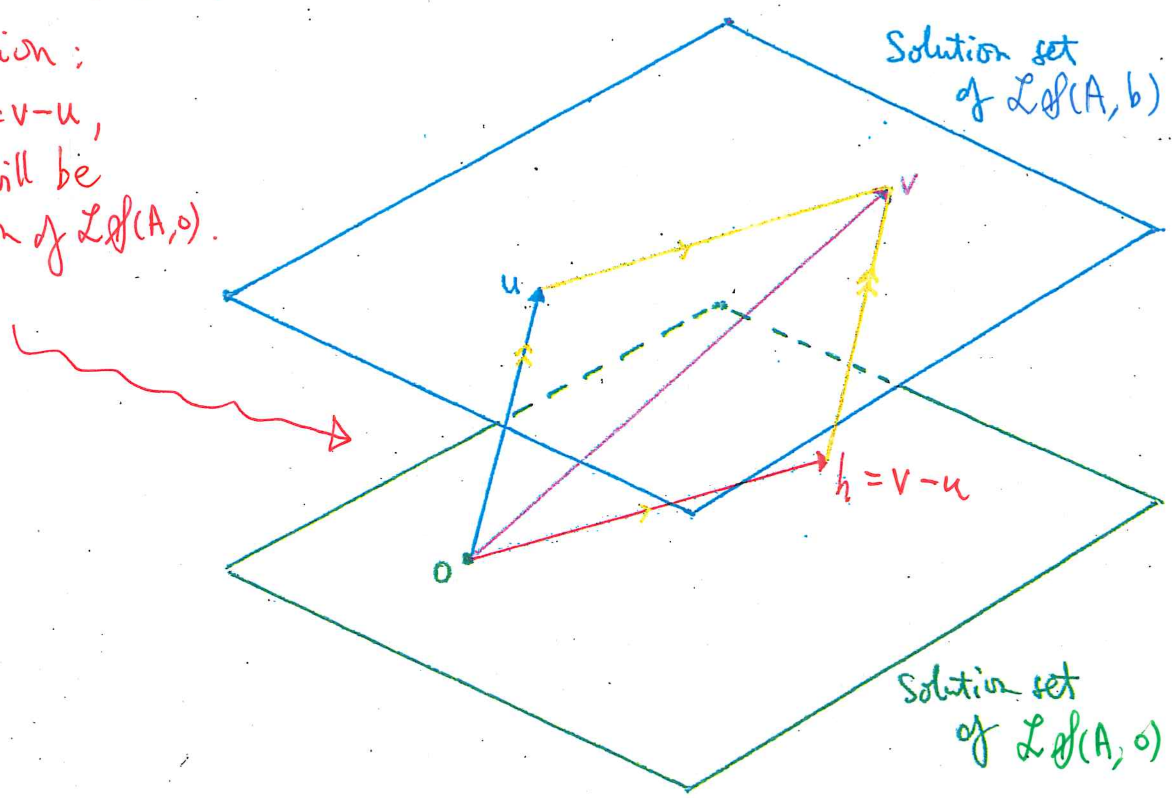


Assumption :

'$x=u$', '$x=v$' are (any) two solutions of $\mathcal{LS}(A,b)$.

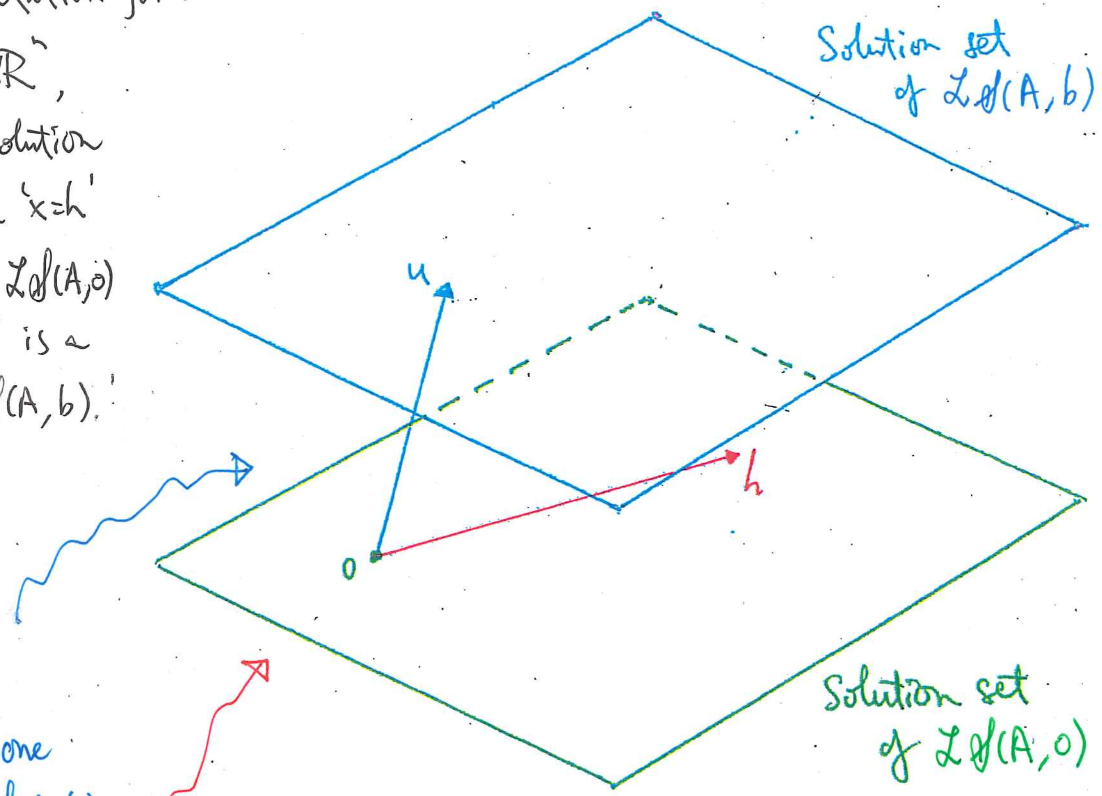
Conclusion :

With $h=v-u$, '$x=h$' will be a solution of $\mathcal{LS}(A,0)$.



Geometric interpretation for :–

'For any $u, h \in \mathbb{R}^n$, if '$x=u$' is a solution of $\mathcal{LS}(A,b)$ and '$x=h$' is a solution of $\mathcal{LS}(A,0)$ then '$x=u+h$' is a solution of $\mathcal{LS}(A,b)$.'



Assumption :

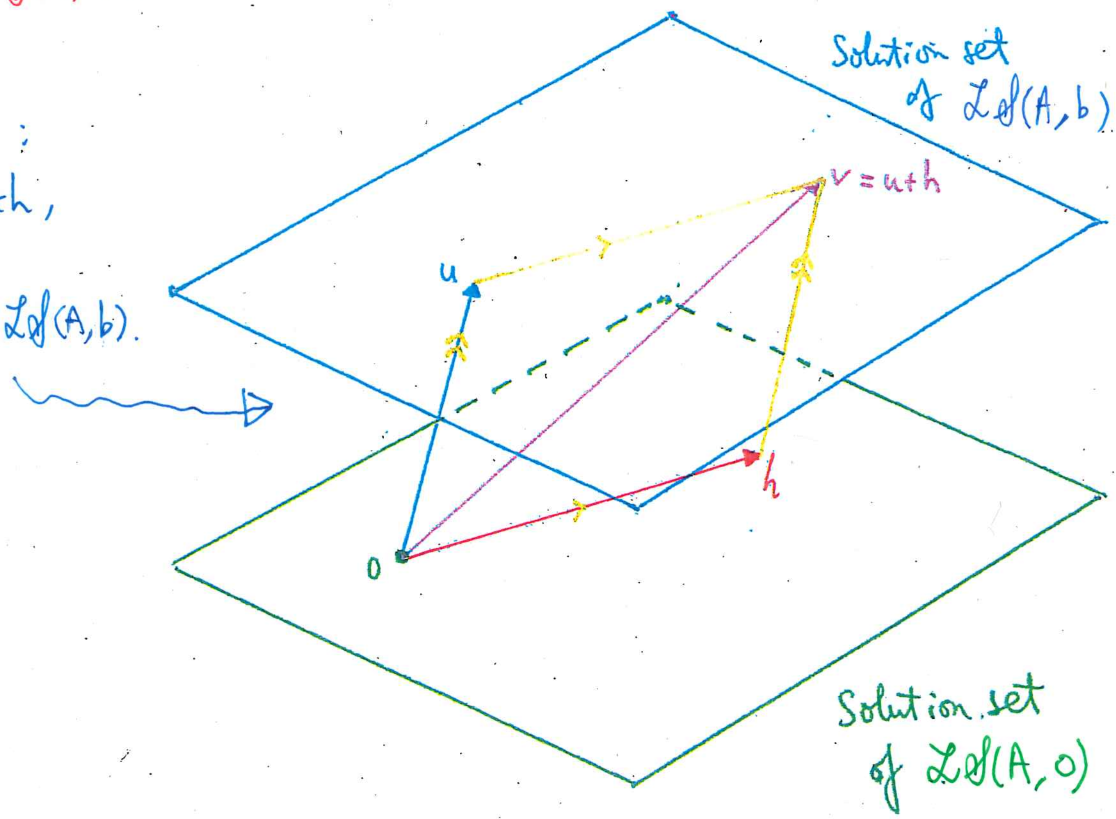
'$x=u$' is (any) one solution of $\mathcal{LS}(A,b)$.

Further assumption :

'$x=h$' is (any) one solution of $\mathcal{LS}(A,0)$.

Conclusion :

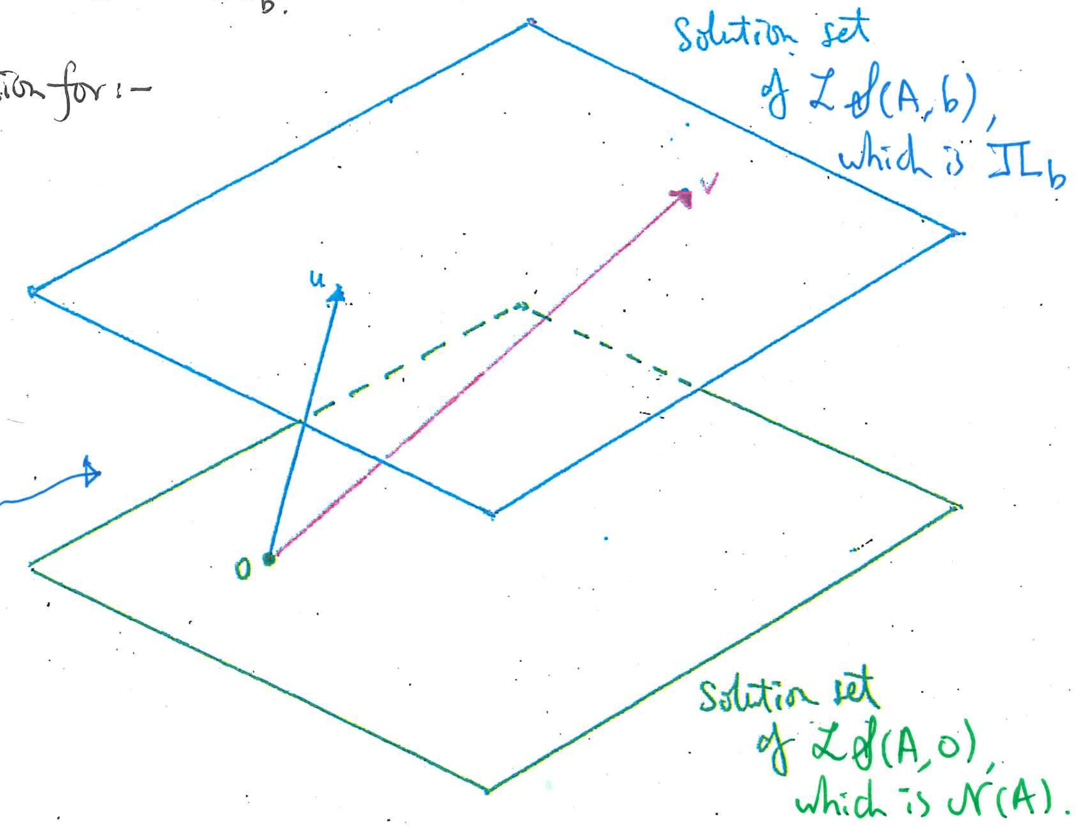
With $v=u+h$, '$x=v$' will be a solution of $\mathcal{LS}(A,b)$.



Given: A is an $(m\times n)$-matrix and b is a vector in $\mathbb{R}^m$.
Further given: u is a vector in Π_b.

Geometric interpretation for:–

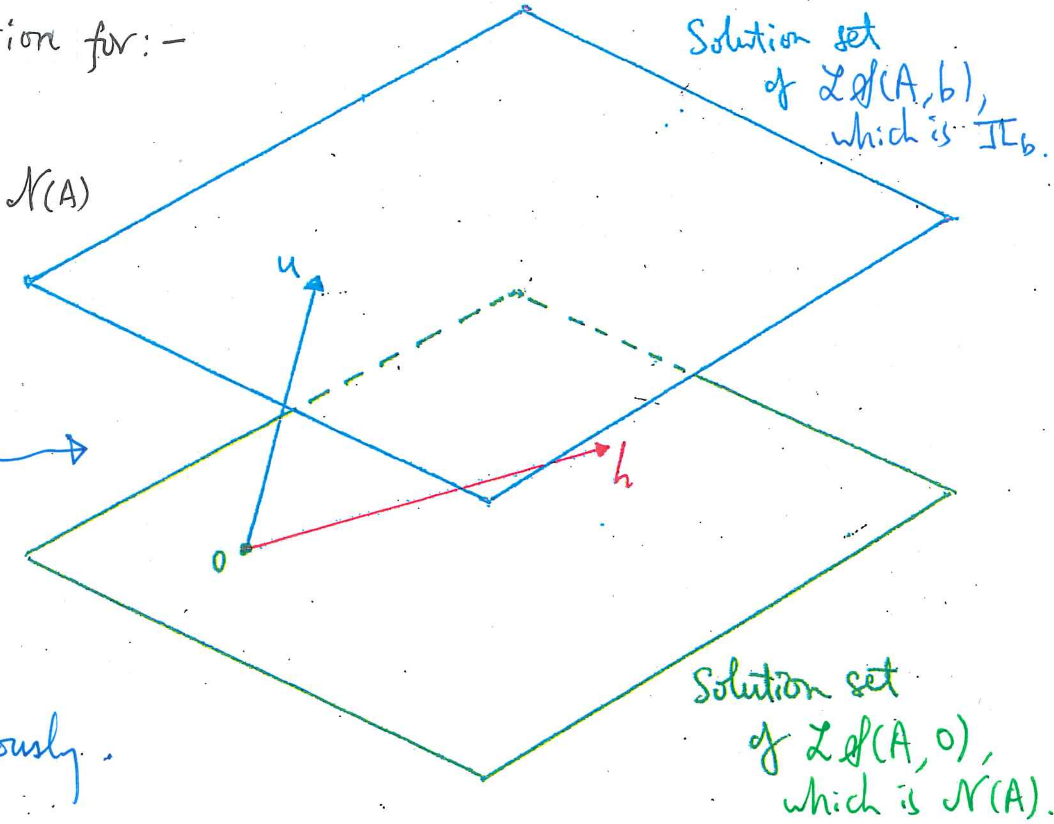
(†) 'For any $v\in\mathbb{R}^n$,
if $v\in\Pi_b$
then $v=u+h$
for some $h\in\mathcal{N}(A)$.'



Assumption:
$v\in\Pi_b$

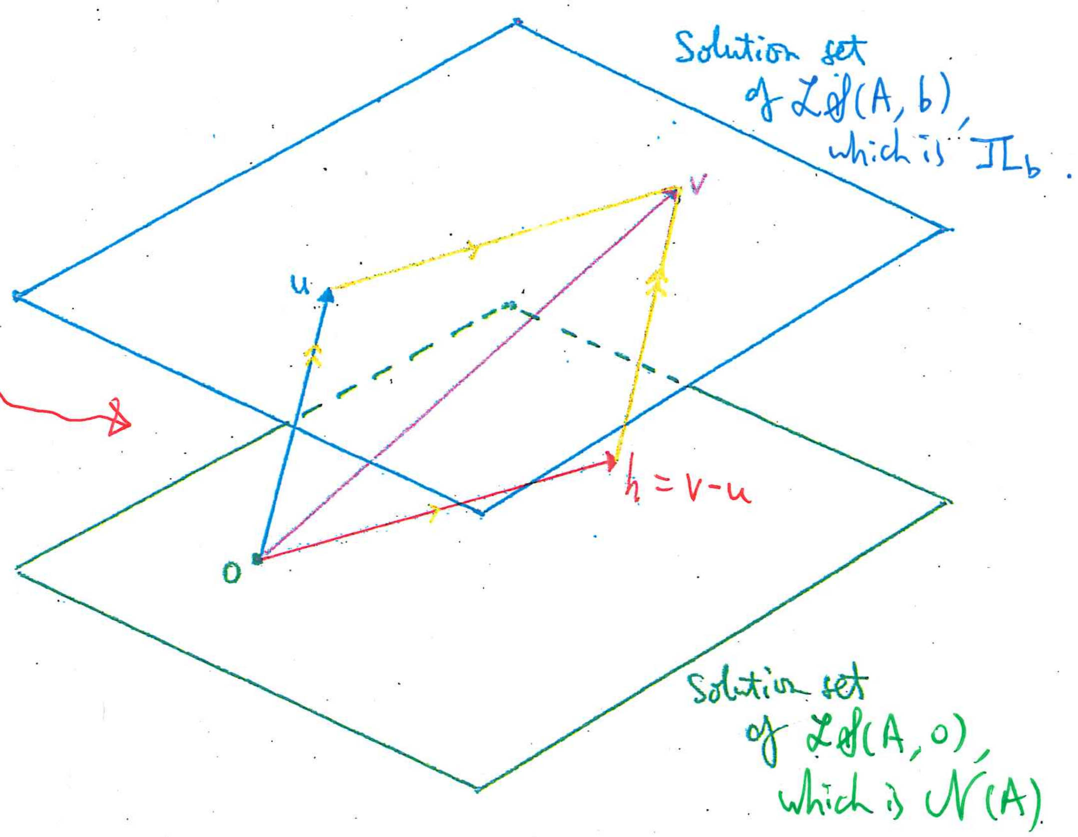
Geometric interpretation for:–

(‡) 'For any $v\in\mathbb{R}^n$,
if $v=u+h$ for some $h\in\mathcal{N}(A)$
then $v\in\Pi_b$.'

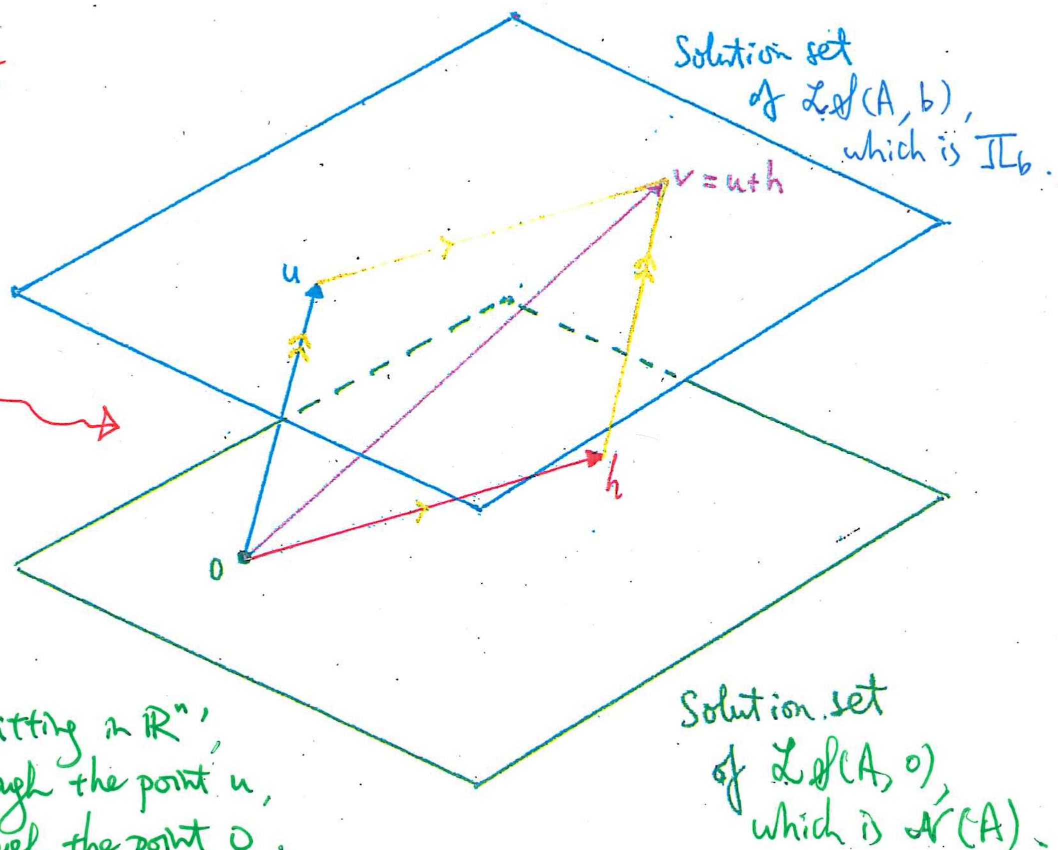


Assumption:
$\begin{cases} v=u+h \\ h\in\mathcal{N}(A)\end{cases}$ simultaneously.

Conclusion:
With $h=v-u$,
it will happen that
$\begin{cases} v=u+h \\ h\in\mathcal{N}(A)\end{cases}$
simultaneously.



Conclusion:
It will happen that
$v\in\Pi_b$.



Consequence of (†), (‡) combined:
Π_b and $\mathcal{N}(A)$ are
two parallel objects 'sitting in $\mathbb{R}^n$',
the former passing through the point u,
the latter passing through the point 0.
Π_b can be obtained from $\mathcal{N}(A)$ by applying to every point of $\mathcal{N}(A)$ a 'translation' by the vector u.