Lecture 12
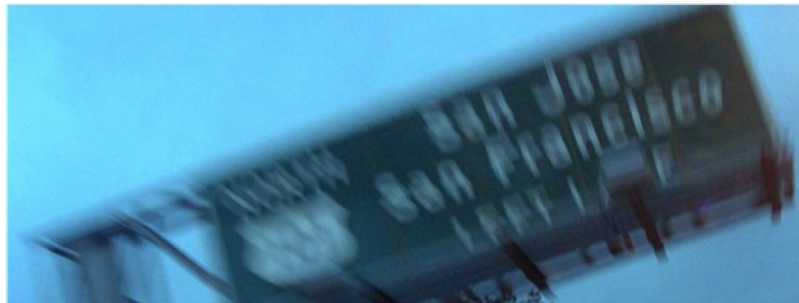
Atmospheric turbulence

Motion Blur

Speeding problem

Image deblurring in the frequency domain:

Mathematical formulation of image blurring

Let g be the observed (blurry) image.

Let f be the original (good) image.

Model g as: $g = H(f) + n$

where H is the degradation function/operator and n is the additive noise.

Assumption on H:

1. H is position invariant:

   Let $g(x,y) = H(f)(x,y)$ and let $\tilde{f}(x,y) := f(x-\alpha, y-\beta)$.
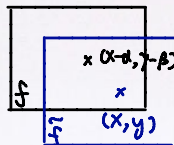
   Then: $H(\tilde{f})(x,y) = g(x-\alpha, y-\beta) = H(f)(x-\alpha, y-\beta)$

2. Linear: $H(f_1 + f_2) = H(f_1) + H(f_2)$

   $H(\alpha f) = \alpha H(f)$ where α is a scalar multiplication.

3. Linearity can be extended to integral:

   $H\left(\iint \alpha(u,v) f(x-u, y-v) \, du \, dv\right) = \iint \alpha(u,v) H(f)(x-u, y-v) \, du \, dv$

With the above assumption, consider an impulse signal:

$$\delta(x,y) = \begin{cases} 1 & \text{if } (x,y) = (0,0) \\ 0 & \text{if } (x,y) \neq (0,0) \end{cases}$$

Then: $f(x,y) = f * \delta(x,y) = \sum_{\alpha=-M/2}^{M/2-1} \sum_{\beta=-N/2}^{N/2-1} f(\alpha,\beta)\,\delta(x-\alpha, y-\beta)$

$\therefore \quad g(x,y) = H(f)(x,y)$

$\qquad = \sum_{\alpha=-M/2}^{M/2-1} \sum_{\beta=-N/2}^{N/2-1} f(\alpha,\beta)\, H(\delta)(x-\alpha, y-\beta) \quad (\text{by linearity and position-invariant})$

$\qquad = \sum_{\alpha=-M/2}^{M/2-1} \sum_{\beta=-N/2}^{N/2-1} f(\alpha,\beta)\, h(x-\alpha, y-\beta) \quad \text{where} \quad h(x,y) = H(\delta)(x,y)$

$\qquad = f * h(x,y)$

$\therefore$ With the above assumption,

$$\text{Degradation/Blur} = \text{Convolution}$$

Remark:

1. h is called the **point spread function**

2. $\therefore g(x,y) = h * f(x,y) + n(x,y)$

   In the frequency domain,

   $$G(u,v) = c\, H(u,v)\, F(u,v) + N(u,v)$$

   $\uparrow$ constant

   $\therefore$ Deblurring can be done by:

   Compute: $F(u,v) \approx \dfrac{G(u,v)}{c\,H(u,v)}$ — from observed image

   — from known degradation

   $\downarrow$

   Obtain: $f(x,y) = DFT^{-1}\big(F(u,v) \big)$

   (Does NOT work very well due to noise!)

Examples of degradation function $H(u,v)$

1. Atmospheric turbulence blur:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

where k = degree of turbulence

$k = 0.0025$ (severe)

$k = 0.001$ (mild)

$k = 0.00025$ (low turbulence)

2. Out of focus blur:

In the frequency domain, define $H(u,v)$ as the DFT of

$$h(x,y) = \begin{cases} 1 & \text{if } x^2+y^2 \leq D_0^2 \\ 0 & \text{otherwise} \end{cases}$$

In some situations, a simple model:

$$H(u,v) = \begin{cases} 1 & \text{if } u^2+v^2 \leq D_0^2 \\ 0 & \text{otherwise} \end{cases} \quad \text{(Usually not accurate)}$$

3. Uniform Linear Motion Blur

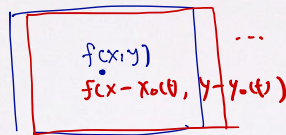Assume $f(x,y)$ undergoes planar motion during acquisition.
(original)

Let $(x_0(t), y_0(t))$ be the motion components in the x- and y-directions
(displacements)
↑
time

Let T be the total exposure time.

The observed image is given by:

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t)) \, dt$$

$f(x,y)$
$f(x - x_0(t), y - y_0(t))$
...

Now, let $G(u,v) = DFT(g)(u,v)$, then:

$$G(u,v) = \frac{1}{N^2} \sum_x \sum_y g(x,y) \, e^{-j\frac{2\pi}{N}(ux+vy)}$$

$$= \frac{1}{N^2} \sum_x \sum_y \int_0^T f(x - x_0(t), y - y_0(t)) \, dt \; e^{-j\frac{2\pi}{N}(ux+vy)}$$

$$= \int_0^T \left(\sum_x \sum_y f(x - x_0(t), y - y_0(t)) \, e^{-j\frac{2\pi}{N}(ux+vy)} \right) dt$$

Recall that $DFT\left(f(x-x_0, y-y_0)\right) = F(u,v)\, e^{-j\frac{2\pi}{N}(u x_0(t) + v\, y_0(t))}$

$F = DFT(f)$

We have:
$$G(u,v) = \int_0^T \left[F(u,v)\, e^{-j\frac{2\pi}{N}(u x_0(t) + v\, y_0(t))} \right] dt$$

$$= F(u,v) \int_0^T e^{-j\frac{2\pi}{N}(u\, x_0(t) + v\, y_0(t))}\, dt$$

$$= F(u,v)\, H(u,v)$$

$\therefore$ Degradation function in the frequency domain is given by:

$$H(u,v) = \int_0^T e^{-j\frac{2\pi}{N}(u x_0(t) + v\, y_0(t))}\, dt$$

(Speeding problem !!)

Example: Suppose the camera is moving left horizontally with a constant speed c. That is, the image at time t is given by:

$$I^*(x,y) = I(x, y - ct)$$

Then: the degradation function is given by:

$$H(u,v) = \int_0^T e^{-j\frac{2\pi}{N}(v(ct))} dt$$

<u>**Remark:**</u> Once the degradation function is known, the original image can be restored by: $IDFT\left(\dfrac{G(u,v)}{H(u,v)}\right)$ (given that there's no noise)

What if there is noise ??

Image deblurring in the frequency domain: (Assume H is known)

Method 1: Direct inverse filtering

Let $T(u,v) = \dfrac{1}{H(u,v) + \varepsilon \, \text{sgn}(H(u,v))}$ ($\text{sgn}(z) = 1$ if $\text{Re}(z) \geq 0$ and $\text{sgn}(z) = -1$ otherwise)

$\underbrace{\phantom{H(u,v) + \varepsilon \, \text{sgn}(H(u,v))}}_{\text{Avoid singularity}}$

Compute $\hat{F}(u,v) = G(u,v) \, T(u,v)$.

Find inverse DFT of $\hat{F}(u,v)$ to get an image $\hat{f}(x,y)$.

Good: Simple

Bad: Boost up noise

$\hat{F}(u,v) = \underset{\underset{H(u,v)F(u,v) + N(u,v)}{\shortparallel}}{G(u,v)} T(u,v) \approx F(u,v) + \dfrac{N(u,v)}{H(u,v) + \varepsilon \, \text{sgn}(H(u,v))}$

<u>Note</u>: $H(u,v)$ is big for (u,v) close to $(0,0)$ (keep low frequencies)
is small for (u,v) far away from $(0,0)$

$\therefore \dfrac{N(u,v)}{H(u,v) + \varepsilon \, \text{sgn}(H(u,v))}$ is big for (u,v) far away from $(0,0)$
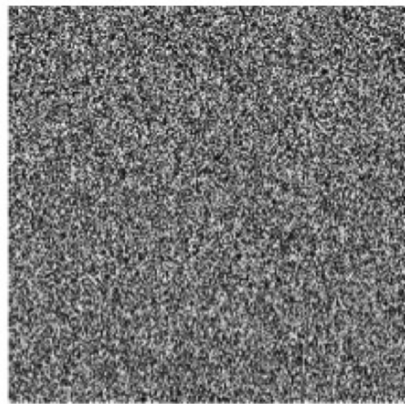
Large gain in high frequencies
⇓
Boost up noises!!

Original   Blurred image   Direct inverse filtering