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1. (1 point) Using: $\lim_{x\to 8} f(x) = 7$ and $\lim_{x\to 8} g(x) = 8$, evaluate

$$\lim_{x\to 8} \frac{f(x)+g(x)}{7f(x)}.$$

Limit = ____
Enter **DNE** if the limit does not exist.

Solution:

$$\lim_{x\to 8} \frac{f(x)+g(x)}{7f(x)} = \frac{\lim_{x\to 8} f(x) + \lim_{x\to 8} g(x)}{7\lim_{x\to 8} f(x)} = \frac{7+8}{7\cdot 7} = 0.306122448979592.$$

---

2. (1 point) Evaluate the limit

$$\lim_{x\to -2} \frac{5x^2-4x+6}{x-4}$$

If the limit does not exist enter DNE.
Limit = __________

Solution:

$$\lim_{x\to -2} \frac{5x^2-4x+6}{x-4} = \frac{\lim_{x\to -2}(5x^2-4x+6)}{\lim_{x\to -2}(x-4)}$$
$$= -\frac{17}{3}$$

---

3. (1 point) Evaluate the limit

$$\lim_{x\to 6} \left(\sqrt{x^2+2} - \frac{x^2+6x}{x} \right)$$

If the limit does not exist enter DNE.
Limit = __________

Solution:

$$\lim_{x\to 6} \left(\sqrt{x^2+2} - \frac{x^2+6x}{x} \right) = \lim_{x\to 6} \sqrt{x^2+2} - \lim_{x\to 6} \frac{x^2+6x}{x}$$
$$= \sqrt{38} - 12$$
$$\approx -5.835585997$$

4. (1 point) Evaluate the limit

$$\lim_{x\to 0} 8\ln x.$$

Enter **DNE** if the limit does not exist.

Limit = _____

Solution:

The limit does not exist. As x approaches 0 from the right, $8\ln x$ becomes unbounded in the negative sense.

---

5. (1 point)

Let $f(x) = \begin{cases} \sqrt{-3-x}+3, & \text{if } x < -4 \\ 3, & \text{if } x = -4 \\ 2x+12, & \text{if } x > -4 \end{cases}$

Calculate the following limits. Enter **DNE** if the limit does not exist.

$\lim_{x\to -4^-} f(x) =$ _______

$\lim_{x\to -4^+} f(x) =$ _______

$\lim_{x\to -4} f(x) =$ _______

Solution:

(a)

$$\lim_{x\to -4^-} f(x) = \lim_{x\to -4^-} (\sqrt{-3-x}+3) \\ = 4$$

(b)

$$\lim_{x\to -4^+} f(x) = \lim_{x\to -4^+} (2x+12) \\ = 4$$

(c)

Because $\lim_{x\to -4^+} f(x) = \lim_{x\to -4^-} f(x)$, $\lim_{x\to -4} f(x)$ exists.

$$\lim_{x\to -4} f(x) = \lim_{x\to -4^+} f(x) = \lim_{x\to -4^-} f(x) = 4.$$

---

6. (1 point) Evaluate the limits.

$$g(x) = \begin{cases} 5x+6 & x < -1 \\ -5 & x = -1 \\ 5x-6 & x > -1 \end{cases}$$

Enter **DNE** if the limit does not exist.

a) $\lim_{x\to -1^-} g(x) =$ _____

b) $\lim_{x\to -1^+} g(x) =$ _____
c) $\lim_{x\to -1} g(x) =$ _____
d) $g(-1) =$ _____
Solution:

(a)

$$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^-} (5x+6) \\ = 1$$

(b)

$$\lim_{x\to -1^+} g(x) = \lim_{x\to -1^+} (5x-6) \\ = -11$$

(c)

$$\lim_{x\to -1^+} g(x) \neq \lim_{x\to -1^-} g(x)$$

So the limit does not exist.
(d)

$$g(-1) = -5$$

---

7. (1 point) Determine the following limits. If a limit *does not exist,* type **DNE**.

$$f(x) = \begin{cases} x-5, & \text{for } x \leq -1 \\ x^2+5, & \text{for } -1 < x \leq 1 \\ 7-x, & \text{for } x > 1 \end{cases}$$

---

1. $\lim_{x\to -1^-} f(x) =$ __________
2. $\lim_{x\to -1^+} f(x) =$ __________
3. $\lim_{x\to -1} f(x) =$ __________
4. $\lim_{x\to 1} f(x) =$ __________
5. $f(-1) =$ __________

Solution:

(1)

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (x-5) \\ = -6$$

(2)

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (x^2+5) \\ = 6$$

(3) $\lim_{x\to -1} f(x)$ does not exist.

(4)

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (7-x)$$
$$= 6$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2+5)$$
$$= 6$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = 6$$

(5)

$$f(-1) = -1-5 = -6$$

---

8. (1 point) Evaluate the limit

$$\lim_{x\to \frac{4}{14}} \frac{14x^2-4x}{|14x-4|}$$

Enter **INF** for ∞, **-INF** for $-\infty$, and **DNE** if the limit does not exist.

Limit = ____________

Solution:

$$\lim_{x\to \frac{4}{14}^+} \frac{14x^2-4x}{|14x-4|} = \lim_{x\to \frac{4}{14}^+} \frac{14x^2-4x}{14x-4}$$
$$= \lim_{x\to \frac{4}{14}^+} x$$
$$= \frac{4}{14}$$

$$\lim_{x\to \frac{4}{14}^-} \frac{14x^2-4x}{|14x-4|} = \lim_{x\to \frac{4}{14}^-} \frac{14x^2-4x}{4-14x}$$
$$= \lim_{x\to \frac{4}{14}^-} (-x)$$
$$= -\frac{4}{14}$$

$$\lim_{x\to \frac{4}{14}^+} \frac{14x^2-4x}{|14x-4|} \neq \lim_{x\to \frac{4}{14}^-} \frac{14x^2-4x}{|14x-4|}$$

So the limit does not exist.

9. (1 point) Evaluate the limit

$$\lim_{x\to 3^-}\left(\frac{1}{x-3}-\frac{1}{|x-3|}\right)$$

Enter **INF** for ∞, **-INF** for $-\infty$, or **DNE** if the limit does not exist (i.e., there is no finite limit and neither ∞ nor $-\infty$ is the limit).

Limit = ____________

Solution:

$$\begin{aligned}\lim_{x\to 3^-}\left(\frac{1}{x-3}-\frac{1}{|x-3|}\right) &= \lim_{x\to 3^-}\left(\frac{1}{x-3}-\frac{1}{3-x}\right)\\ &= \lim_{x\to 3^-}\left(\frac{2}{x-3}\right)\\ &= -\infty\end{aligned}$$

---

10. (1 point)

Find $\lim_{x\to\infty} f(x)$ if $\dfrac{4x-1}{x} < f(x) < \dfrac{4x^2+3x}{x^2}$ for all $x > 5$.

_____

Solution:

By Squeeze Theorem, since

$$\lim_{x\to\infty}\left(\frac{4x-1}{x}\right) = \lim_{x\to\infty}\left(\frac{4x^2+3x}{x^2}\right) = 4,$$

we must have

$$\lim_{x\to\infty} f(x) = 4.$$

---

11. (1 point) Use the Squeeze Theorem to evaluate the limit $\lim_{x\to 8} f(x)$, if

$$16x-64 \le f(x) \le x^2 \qquad \text{on } [6,10].$$

Enter **DNE** if the limit does not exist.

Limit = _____

Solution:

$$\lim_{x\to 8}(16x-64) = 16(8)-64 = 64.$$

$$\lim_{x\to 8} x^2 = (8)^2 = 64.$$

By the Squeeze theorem, $\lim_{x\to 8} f(x) = 64$.

---

12. (1 point) Use the Squeeze Theorem to evaluate the limit

$$\lim_{x\to 0} \sin x \cos\left(\frac{1}{x^4}\right)$$

Enter **DNE** if the limit does not exist.

Limit = ____

Solution:

Regardless of the value of $x \neq 0$,

$$-1 \leq \cos\left(\frac{1}{x^4}\right) \leq 1$$

We consider the absolute value of this sequence directly.

$$|\sin x \cos(\frac{1}{x^4})| \leq |\sin x|$$

which means that

$$-|\sin x| \leq |\sin x \cos(\frac{1}{x^4})| \leq |\sin x|$$

By the Squeeze Theorem, since $\lim_{x\to 0} \sin x = 0$, we must have

$$\lim_{x\to 0} |\sin x \cos\left(\frac{1}{x^4}\right)| = 0.$$

So we have that

$$\lim_{x\to 0} \sin x \cos\left(\frac{1}{x^4}\right) = 0.$$

---

13. (1 point) Let

$$f(x) = \frac{x^2+5}{x^2-4}.$$

Find the indicated one-sided limits of f.

NOTE: Remember that you use **INF** for ∞ and **-INF** for $-\infty$.

You should also sketch a graph of $y = f(x)$, including vertical and horizontal asymptotes.

$\lim_{x\to -2^-} f(x)$ = ____________

$\lim_{x\to -2^+} f(x)$ = ____________

$\lim_{x\to 2^-} f(x)$ = ____________

$\lim_{x\to 2^+} f(x)$ = ____________

$\lim_{x\to -\infty} f(x)$ = ____________

$\lim_{x\to \infty} f(x)$ = ____________

Solution:

(a)

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \frac{x^2+5}{x^2-4} = \lim_{x\to -2^-} \frac{x^2+5}{(x-2)(x+2)}$$
$$= \infty$$

(b)

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{x^2+5}{x^2-4} = \lim_{x\to -2^+} \frac{x^2+5}{(x-2)(x+2)}$$
$$= -\infty$$

(c)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2+5}{x^2-4} = \lim_{x\to 2^-} \frac{x^2+5}{(x-2)(x+2)}$$
$$= -\infty$$

(d)

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x^2+5}{x^2-4} = \lim_{x\to 2^+} \frac{x^2+5}{(x-2)(x+2)}$$
$$= \infty$$

(e)

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{x^2+5}{x^2-4}$$
$$= 1$$

(f)

$$\lim_{x\to \infty} f(x) = \lim_{x\to \infty} \frac{x^2+5}{x^2-4}$$
$$= 1$$

---

14. (1 point)
Observe that the function is continuous at each member of its domain.
State the domain as an interval or union of disjoint intervals.

$$f(x) = x^2 + \sqrt{2x-1}$$

Domain: ________
(Use 'U' for union and 'inf', '-inf' for ∞, $-\infty$ if necessary).
Solution:

Solving

$$2x-1 \geq 0$$

gives the domain: $[\frac{1}{2}, \infty)$

15. (1 point)

Part 1: Evaluate the limit

Suppose

$$f(x) = \begin{cases} x^2+10x+30, & x < -5, \\ -2, & x = -5, \\ -x^2-10x-20, & x > -5. \end{cases}$$

Evaluate the following limits (with a function of x in the first answer blank and the value of the limit in the second answer blank) and evaluate the function.

a. $\lim_{x\to-5^-} f(x) = \lim_{x\to-5^-}$ ______ = ____.

b. $\lim_{x\to-5^+} f(x) = \lim_{x\to-5^+}$ ______ = ____.

c. $f(-5) =$ ____.

Part 2: Continuity

Solution:

Part 1

(a)

$$\lim_{x\to-5^-} f(x) = \lim_{x\to-5^-} (x^2+10x+30) = 5$$

(b)

$$\lim_{x\to-5^+} f(x) = \lim_{x\to-5^+} (-x^2-10x-20) = 5$$

(c)

$$f(-5) = -2$$

Part 2.

The function is not continuous at -5.

---

16. (1 point)

a. [choose/true/false] If $\lim_{x\to1^-} f(x) = 5$, then $\lim_{x\to1} f(x) = 5$.

b. [choose/true/false] If $\lim_{x\to1^-} f(x) = 5$, then $\lim_{x\to1^+} f(x) = 5$.

c. [choose/true/false] If $\lim_{x\to1} f(x) = 5$, then $\lim_{x\to1^-} f(x) = 5$.

d. [choose/true/false] If $\lim_{x\to1} f(x) = 5$, then $\lim_{x\to1^+} f(x) = 5$.

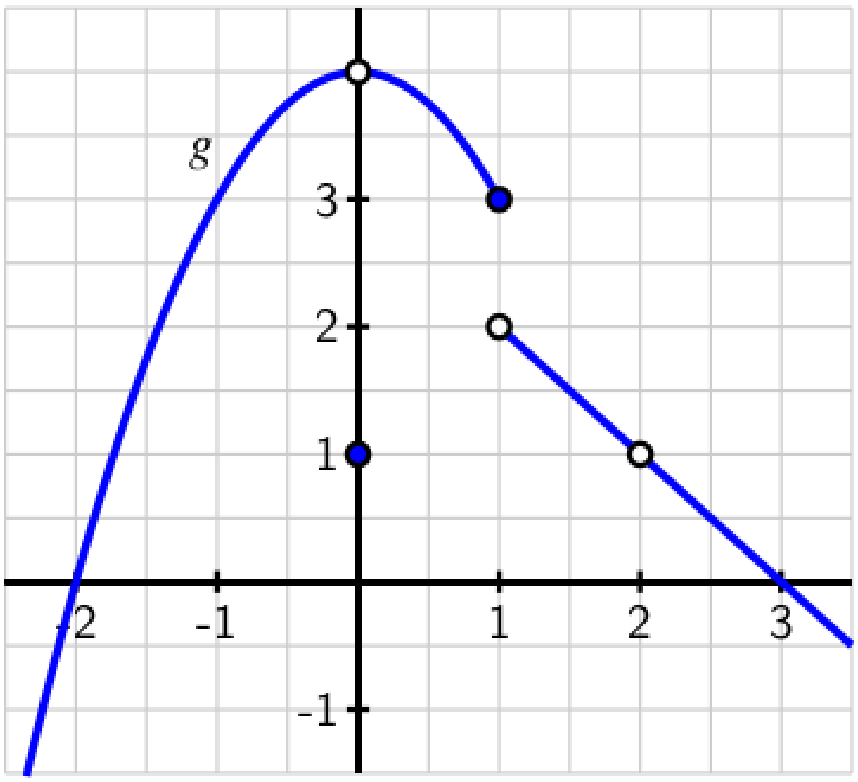
e. Select all true statements. Assume that all the limits are all taken at the same point.

- A. If the left- and right-hand limits both exist and are equal, then the two-sided limit exists.
- B. If the right-hand limit exists, then the two-sided limit exists.
- C. If the left-hand limit exists, then the two-sided limit exists.
- D. If the two-sided limit exists, then the left- and right-hand limits both exist and are equal.

Solution:
(1) False
(2) False
(3) True
(4) True
(5) AD

---

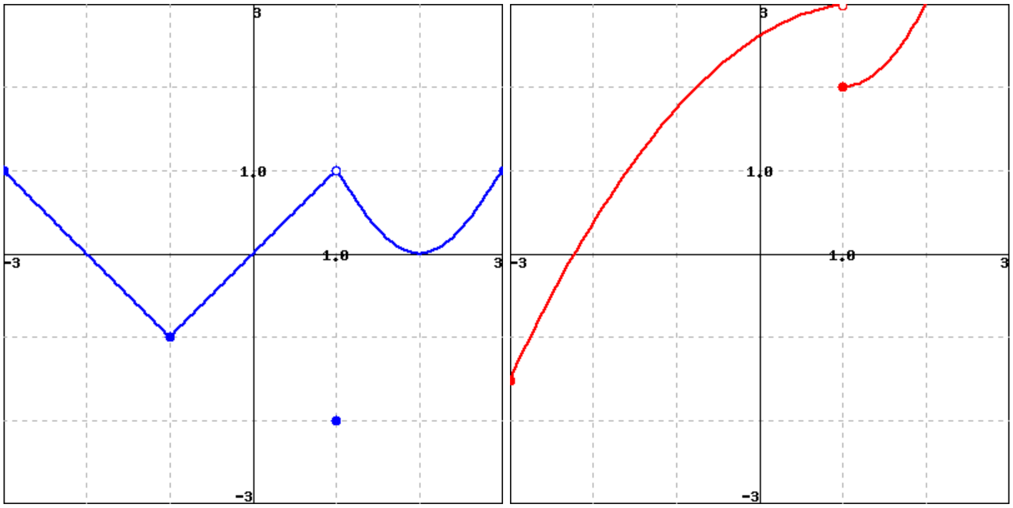
17. (1 point) Suppose that g is the function given by the graph below. Use the graph to fill in the blanks in the following sentences.



As x gets closer and closer (but not equal) to -1, $g(x)$ gets as close as we want to ___.
As x gets closer and closer (but not equal) to 0, $g(x)$ gets as close as we want to ___.
As x gets closer and closer (but not equal) to 2, $g(x)$ gets as close as we want to ___.
Solution:
As x gets closer and closer (but not equal) to -1, $g(x)$ gets as close as we want to 3, because the function in this neighborhood is a continuous curve that passes through the point $(-1,3)$. As x gets closer and closer (but not equal) to 0, $g(x)$ gets as close as we want to 4, because the values of the function on both sides of $x = 0$ are approaching 4, even though the value of $g(0)$ is different. As x gets closer and closer (but not equal) to 2, $g(x)$ gets as close as we want to 1, because the values of the function on both sides of $x = 2$ are approaching 1, even though the value of $g(2)$ is undefined.

---

18. (1 point) Use the given graphs of the function f (left, in blue) and g (right, in red) to find the following limits:



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1. $\lim_{x\to 1}[f(x)+g(x)] =$ ________ help (limits)
2. $\lim_{x\to 2}[f(x)+g(x)] =$ ________
3. $\lim_{x\to 0} f(x)g(x) =$ ________
4. $\lim_{x\to 0} \dfrac{f(x)}{g(x)} =$ ________
5. $\lim_{x\to -1} \sqrt{3+f(x)} =$ ________

Note: *You can click on the graphs to enlarge the images.*

Solution:
1. DNE
2. 3
3. 0
4. 0
5. $\sqrt{2}$

As x gets closer and closer (but not equal) to 1, the left limit of $g(x)$ does not equal to the right limit of that. Then the limit does not exist. As x gets closer and closer to 2, $g(x)$ gets as close as we want to 3 and $f(x)$ gets as close as we want to 0. As x gets closer and closer to 0, $g(x)$ gets as close as we want to a constant belongs to $[2,3]$ and $f(x)$ gets as close as we want to 0. As x gets closer and closer to -1, $f(x)$ gets as close as we want to -1.