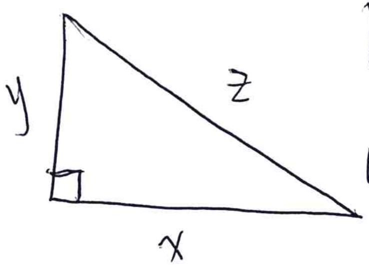

Problem: For all the right triangle which have perimeter P. determine when it gets the biggest area.

Sol: Lagrange function.

$$F = \frac{1}{2}xy + \lambda(x+y+z-P) + \mu(x^2+y^2-z^2)$$

$$\frac{\partial F}{\partial x} = \frac{1}{2}y + \lambda + 2\mu x = 0 \quad (1)$$

$$\frac{\partial F}{\partial y} = \frac{1}{2}x + \lambda + 2\mu y = 0 \quad (2)$$

$$\frac{\partial F}{\partial z} = \lambda - 2\mu z = 0 \quad (3)$$

$$x+y+z=P \quad (4)$$

$$x^2+y^2=z^2 \quad (5)$$

① − ②

$\Rightarrow (\frac{1}{2} - 2\mu)(y - x) = 0$ ⑥

then there are 2 cases.

first we suppose $\mu = \frac{1}{4}$ ($\frac{1}{2} - 2\mu = 0$).

① + ②

$\Rightarrow x + y + 2\lambda = 0$

but ③ implies $z = 2\lambda$.

$\Rightarrow \quad x + y + z = 0 \quad (P \neq 0)$

contradiction!

we must have $x = y$ in ⑥

$z^2 = x^2 + y^2 = x^2 + x^2 = 2x^2$

$\Rightarrow z = \sqrt{2}\, x$ (by $x, y, z > 0$)

substitute them in ④ we get

$$2x + \sqrt{2}\, x = P$$

$$\Rightarrow \quad x = y = \frac{P}{2+\sqrt{2}}$$

$$z = \frac{\sqrt{2}\, P}{2+\sqrt{2}}$$

that's the time when it gets the largest area!