MATH 2010A/B Advanced Calculus I
(2014-2015, First Term)
Homework 1
Suggested Solution

Exercises 12.1

11. The circle $x^2 + y^2 = 16$ in the xy-plane.

13. The ellipse formed by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = y$.

15. The parabola $y = x^2$ in the the xy-plane.

17. (a) The first quadrant of the xy-plane
    (b) The fourth quadrant of the xy-plane

19. (a) The solid ball of radius 1 centered at the origin
    (b) The exterior of the sphere of radius 1 centered at the origin

21. (a) The solid enclosed between the sphere of radius 1 and radius 2 centered at the origin
    (b) The solid upper hemisphere of radius 1 centered at the origin

23. (a) The region on or inside the parabola $y = x^2$ in the xy-plane and all points above this region.
    (b) The region on or to the left of the parabola $x = y^2$ in the xy-plane and all points above it that are 2 units or less away from the xy-plane.

27. (a) $z = 1$
    (b) $x = 3$
    (c) $y = -1$

29. (a) $x^2 + (y-2)^2 = 4, z = 0$
    (b) $(y-2)^2 + z^2 = 4, x = 0$
    (c) $x^2 + z^2 = 4, y = 2$

31. (a) $y = 3, z = -1$
    (b) $x = 1, z = -1$
    (c) $x = 1, y = 3$

33. $x^2 + y^2 + z^2 = 25, z = 3 \Rightarrow x^2 + y^2 = 16$ in the plane $z = 3$

35. $0 \leq z \leq 1$

37. $z \leq 0$

57. $2x^2 + 2y^2 + 2z^2 + x + y + z = 9$

$\Rightarrow x^2 + \frac{1}{2}x + y^2 + \frac{1}{2}y + z^2 + \frac{1}{2}z = \frac{9}{2}$

$\Rightarrow \left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) + \left(y^2 + \frac{1}{2}y + \frac{1}{16}\right) + \left(z^2 + \frac{1}{2}z + \frac{1}{16}\right) = \frac{9}{2} + \frac{3}{16}$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 + \left(z + \frac{1}{4}\right)^2 = \left(\frac{5\sqrt{3}}{4}\right)^2$

$\Rightarrow$ the center is at $(-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4})$ and the radius is $\frac{5\sqrt{3}}{4}$

59. (a) the distance between (x, y, z) and $(x, 0, 0)$ is $\sqrt{y^2 + z^2}$
    (b) the distance between (x, y, z) and $(0, y, 0)$ is $\sqrt{x^2 + z^2}$
    (c) the distance between (x, y, z) and $(0, 0, z)$ is $\sqrt{x^2 + y^2}$

60. (a) the distance between (x, y, z) and $(x, y, 0)$ is z
    (b) the distance between (x, y, z) and $(0, y, z)$ is x
    (c) the distance between (x, y, z) and $(x, 0, z)$ is y

Exercises 12.2

7. (a) $\frac{3}{5}\mathbf{u} = \left\langle \frac{9}{5}, -\frac{6}{5} \right\rangle$

$\frac{4}{5}\mathbf{v} = \left\langle -\frac{8}{5}, 4 \right\rangle$

$\frac{3}{5}\mathbf{u} + \frac{4}{5}\mathbf{v} = \left\langle \frac{9}{5} + \left(-\frac{8}{5}\right), -\frac{6}{5} + 4 \right\rangle = \left\langle \frac{1}{5}, \frac{14}{5} \right\rangle$

(b) $\sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{14}{5}\right)^2} = \frac{\sqrt{197}}{5}$

12. $\overrightarrow{AB} =< 2 - 1, 0 - (-1) >=< 1, 1 >$

$\overrightarrow{CD} =< -2 - (-1), 2 - 3 >=< -1, -1 >$

$\overrightarrow{AB} + \overrightarrow{CD} =< 0, 0 >$

19. $\overrightarrow{AB} = (-10 - (-7))\mathbf{i} + (8 - (-8))\mathbf{j} + (1 - 1)\mathbf{k} = -3\mathbf{i} + 16\mathbf{j}$

21. $5\mathbf{u} - \mathbf{v} = 5\langle 1, 1, -1 \rangle - \langle 2, 0, 3 \rangle = \langle 3, 5, -8 \rangle = 3\mathbf{i} + 5\mathbf{j} - 8\mathbf{k}$

33. $|\mathbf{v}| = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$; $\frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{13}\mathbf{v} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{k})$

$\Rightarrow$ the desired vector is $\frac{7}{13}(12\mathbf{i} - 5\mathbf{k})$

34. $|\mathbf{v}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$; $\frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}$
$\Rightarrow$ the desired vector is $-3\left(\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} - \frac{1}{\sqrt{3}}\mathbf{k}\right) = -\sqrt{3}\mathbf{i} + \sqrt{3}\mathbf{j} + \sqrt{3}\mathbf{k}$

41. $2\mathbf{i} + \mathbf{j} = a(\mathbf{i} + \mathbf{j}) + b(\mathbf{i} - \mathbf{j})$
$\Rightarrow a + b = 2$ and $a - b = 1$
$\Rightarrow 2a = 3 \Rightarrow a = \frac{3}{2}$ and $b = a - 1 = \frac{1}{2}$

42. $\mathbf{i} - 2\mathbf{j} = a(2\mathbf{i} + 3\mathbf{j}) + b(\mathbf{i} - \mathbf{j}) = (2a + b)\mathbf{i} + (3a + b)\mathbf{j}$
$\Rightarrow 2a + b = 1$ and $3a + b = -2$
$\Rightarrow a = -3$ and $b = 1 - 2a = 7$
$\Rightarrow \mathbf{u}_1 = a(2\mathbf{i} + 3\mathbf{j}) = -6\mathbf{i} - 9\mathbf{j}$ and $\mathbf{u}_2 = b(\mathbf{i} + \mathbf{j}) = 7\mathbf{i} + 7\mathbf{j}$

56. Let $\mathbf{u}$ be any unit vector in the plane. If $\mathbf{u}$ is positioned so that its initial point is at the origin and terminal point is at (x, y), then $\mathbf{u}$ makes an angle θ with $\mathbf{i}$, measured in the counter-clockwise direction. Since $|\mathbf{u}| = 1$, we have that $x = \cos\theta$ and $y = \sin\theta$. Thus $\mathbf{u} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$. Since $\mathbf{u}$ was assumed to be any unit vector in the plane, this holds for every unit vector in the plane.

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