THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050C Mathematical Analysis I

Tutorial 4 (February 15)

The following were discussed in the tutorial this week:

1 Sequences and their limits

Definition 1.1. *A sequence $X = (x_n)$ in $\mathbb{R}$ is said to be converge to $x \in \mathbb{R}$, or x is said to be a limit of (x_n), if for every $\varepsilon > 0$ there exists a natural number $K(\varepsilon)$ such that for all $n \geq K(\varepsilon)$, the terms x_n satisfy $|x_n - x| < \varepsilon$.*

1.1 Procedure.

1. *Let $\varepsilon > 0$ be given. (ε is arbitrary, but cannot be changed once fixed.)*
2. *Find a useful estimate for $|x_n - x|$.*
3. *Find $K(\varepsilon) \in \mathbb{N}$ such that the estimate in 2 is less than ε.*
4. *Complete the proof.*

Example 1.1. *Use the definition of the limit of a sequence to establish the following limit.* $\lim \left(\frac{n^2 - n}{2n^2 + 3} \right) = \frac{1}{2}$

2 Limit Theorems

2.1 Squeeze Theorem. *Suppose $X = (x_n)$, $Y = (y_n)$ and $Z = (z_n)$ are sequences of real numbers such that*

$$x_n \leq y_n \leq z_n \quad \text{for all } n \in \mathbb{N},$$

and that $\lim(x_n) = \lim(z_n)$. Then $Y = (y_n)$ is convergent and

$$\lim(x_n) = \lim(y_n) = \lim(z_n).$$

Example 2.1. *Use the Squeeze Theorem to determine the limits of the following.*

(a) $\left((n!)^{1/n^2}\right)$

(b) $(3^n/n!))$

3 Classwork

1. Use the definition of the limit of a sequence to establish the following limit:
   $\lim\left(\frac{5n^2+2n+1}{3n^2+n+2}\right)=\frac{5}{3}$.
2. Use the Squeeze Theorem to determine the following limit:
   $\lim\left(\sqrt[n]{3^n+|2\sin(n^n)|^n}\right)$.