THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050C Mathematical Analysis I

Tutorial 2 (January 23)

The following were discussed in the tutorial this week:

1 Absolute Value and the Real Line

Example 1.1. *Sketch the graph of the equation* $y = |x| - |x - 1|$.

2 The Completeness Property of $\mathbb{R}$

Definition 2.1. *Let* S *be a nonempty subset of* $\mathbb{R}$.

(a) Suppose S *is bounded above. Then* $u \in \mathbb{R}$ *is said to be a* ***supremum*** *of* S *if*

(i) u *is an upper bound of* S *(that is,* $s \le u$ *for all* $s \in S$*);*

(ii) if v *is any upper bound of* S*, then* $u \le v$.

Here (ii) is equivalent to

(ii)' if $v < u$*, then there exists* $s_v \in S$ *such that* $v < s_v$.

(b) Suppose S *is bounded below. Then* $w \in \mathbb{R}$ *is said to be an* ***infimum*** *of* S *if*

(i) w *is a lower bound of* S *(that is,* $w \le s$ *for all* $s \in S$*);*

(ii) if v *is any lower bound of* S*, then* $v \le w$.

Here (ii) is equivalent to

(ii)" if $w < v$*, then there exists* $s_v \in S$ *such that* $s_v < v$.

Remark. *1. Supremum and infimum may not be elements of* S.

2. u *and* w *above are unique and we write* $\sup S = u$, $\inf S = w$.

Example 2.1. *Let* $S_1 := \{x \in \mathbb{R} : x \ge 0\}$*. Show that the set* S_1 *has lower bounds, but no upper bounds. Show that* $\inf S_1 = 0$.

Example 2.2. *Find the infimum and supremum, if they exist, of the set* $A := \{x \in \mathbb{R} : 1/x < x\}$*. Justify your answers.*

3 Classwork

Let $A := \{x \in \mathbb{R} : |x+2| + |x-1| > 5\} \cap \{x \in \mathbb{R} : x \leq 0\}$.

(a) What are the elements of the set A?

(b) Determine whether A is bounded above or bounded below.

(c) Find $\sup A$ and $\inf A$, if they exist.