MATH 2050C Mathematical Analysis I
2018-19 Term 2

Solution to Problem Set 4

2.5-8

Suppose $\cap_{n=1}^{\infty} J_n \neq \emptyset$ and $x \in \cap_{n=1}^{\infty} J_n$. Thus $x \in J_n, \forall n$ and $x > 0$. By Archimedean property, there exists some $N \in \mathbb{N}$ satisfying $Nx > 1$. Thus $x > \frac{1}{N}$ and $x \notin J_N$. Contradiction.

3.1-5(c)

For arbitrary $\varepsilon > 0$, if $K > \frac{13}{4\varepsilon}$, then for all $n > K$,

$$\left| \frac{3n+1}{2n+5} - \frac{3}{2} \right| = \frac{13}{2(2n+5)} < \frac{13}{4n} < \frac{13}{4K} < \varepsilon.$$

3.1-6(c)

For arbitrary $\varepsilon > 0$, if $K > \frac{1}{\varepsilon^2}$, then for all $n > K$,

$$\left| \frac{\sqrt{n}}{n+1} \right| < \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} < \frac{1}{\sqrt{K}} < \frac{1}{\sqrt{\frac{1}{\varepsilon^2}}} = \varepsilon.$$

3.1-7

(a) For arbitrary $\varepsilon > 0$, if $K > \exp \frac{1}{\varepsilon}$, then for all $n > K$,

$$\left| \frac{1}{\ln(n+1)} \right| < \frac{1}{\ln n} < \frac{1}{\ln K} < \frac{1}{\ln \exp \frac{1}{\varepsilon}} = \varepsilon.$$

(b) (i) For $\varepsilon = \frac{1}{2}$, set $K_1 = 3^2 > \exp 2$.

(ii) For $\varepsilon = \frac{1}{10}$, set $K_2 = 3^{10} > \exp 10$.

3.1-11

Note that $\frac{1}{n} \leq 1, \forall n \in \mathbb{N}$. For arbitrary $\varepsilon > 0$, if $K > \frac{1}{\varepsilon}$, then for all $n > K$,

$$\left| \frac{1}{n} - \frac{1}{n+1} \right| = \frac{1}{n^2 + n} < \frac{1}{n^2} \leq \frac{1}{n} < \frac{1}{K} < \varepsilon.$$