MATH 2050C Mathematical Analysis I
2018-19 Term 2

Solution to Problem Set 2

2.2-10

(a) $|x-1| > |x+1| \iff (x-1)^2 > (x+1)^2 \iff x < 0$.
The solution set is $(-\infty, 0)$.

(b) If $x \le -1$, $|x| + |x+1| < 2 \iff -2x - 1 < 2 \iff x > -\frac{3}{2}$.
In this case, the solution set is $(-\frac{3}{2}, -1]$.
If $-1 < x < 0$, $|x| + |x+1| < 2 \iff 1 < 2 \iff -1 < x < 0$.
In this case, the solution set is $(-1, 0)$.
If $x \ge 0$, $|x| + |x+1| < 2 \iff 2x + 1 < 2 \iff x < \frac{1}{2}$.
In this case, the solution set is $[0, \frac{1}{2})$.
Combine all three cases and the solution set is $(-\frac{3}{2}, \frac{1}{2})$

2.2-12

$$|x+2| + |x-1| = \begin{cases} -2x-1 & x < -2; \\ 3 & -2 \le x \le 1; \\ 2x+1 & x > 1. \end{cases}$$

$$\begin{aligned} & 4 < |x+2| + |x-1| < 5 \\ \iff & \begin{cases} 4 < -2x - 1 < 5, \\ x < -2; \end{cases} \quad \text{or} \begin{cases} 4 < 3 < 5, \\ -2 \le x \le 1; \end{cases} \quad \text{or} \begin{cases} 4 < 2x + 1 < 5, \\ x > 1. \end{cases} \\ \iff & -3 < x < -\frac{5}{2} \text{ or } \frac{3}{2} < x < 2. \end{aligned}$$

The solution set is $(-3, -\frac{5}{2}) \cup (\frac{3}{2}, 2)$.

2.3-7

S is bounded above since it has some upper bound which it contains. Denote the contained upper bound as u_0. From the definition of supremum, $\sup S \leq u_0$, since u_0 is an upper bound. On the other hand, that $u_0 \in S$ implies that $u_0 \leq \sup S$ because $s \leq \sup S, \forall s \in S$. Combine these two inequalities, $u_0 = \sup S$.

2.3-14

From the definition, a lower bound w of S is the infimum of S if and only if $v \leq w$ for any lower bound v of S. In the following, we prove by contradiction. Assume $w = \inf S$. Suppose that there exists some ε_0 so that $t \geq w + \varepsilon_0, \forall t \in S$. Thus $w + \varepsilon_0$ is a lower bound by the definition, contradicting that $v \leq w$ for any lower bound v of S.
Assume the condition hold. Suppose that w is not the infimum. There exists lower bound v with $w < v$ and $t \leq v, \forall t \in S$. Take $\varepsilon = v - w > 0$. From the condition, $t_0 < w + \varepsilon = v$ for some $t_0 \in S$, contradicting that v is a lower bound.