THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 6021 (First term, 2020-21)
Essay and Presentation

The course assessment of MATH 6021 will consist of one in-class presentations (which everyone must attend) and a written mathematical essay on selected topics. Note the following important dates:

- *In-class presentation (via ZOOM)*: **December 7, 2020**, 9:30AM-12:15PM
- *Essay submission deadline (by email)*: **December 14, 2020** at 11:59PM

Presentations and Essay

Presentations: Every student should give his/her own presentation in English. The presentations will be evaluated in terms of content, clarity and pedagogy. You should give some motivation and introductory/background material about the chosen topic, highlight some of the most interesting theorems related, and give some connections to other areas if possible. The presentations should be kept to be strictly less than 15 minutes for the time constraint.

Written essay: You should write up in LaTeX a mathematical article about the your selected topic. The essay should be at least 5 pages long. Each student is responsible for writing up his/her own essay and send it to the instructor via email by the deadline stated above. More specifically, the essay should comprise of precise statements of the problems you have studied, some of the background of the mathematics involved, and at least a sketch of the ideas of the proofs of some major theorems. References should be supplied at the end of the report (which does not count towards the 5-page requirement).

Some suggested topics

You are welcomed to choose a topic of your own to work on. Below are some suggestions (which will be constantly updated until the end of the semester)

- There is an approach to the Bersntein Problem in two-dimensional using complex analytic techniques. L. Nirenberg conjectured, around 1950, that the Gauss map of a complete minimal surface of $\mathbb{R}^3$ could not omit a neighbourhood of a point unless M was a plane. Nirenberg's conjecture, a generalization of Bernstein's theorem, can also be viewed as a weak form of Picard's theorem in complex analysis. *Ref: R. Osserman, "Proof of a conjecture of Nirenberg", Communication on Pure and Applied Mathematics, vol. 12, 229-232 (1959).*
- Study the interior and boundary branch points that may arise in Douglas-Rado's approach to the Plateau's Problem. *Ref: R. Gulliver, "Regularity of Minimizing Surfaces of Prescribed Mean Curvature", Annals of Mathematics 97, 275-305 (1973), B. White, "Classical area minimizing surfaces with real-analytic boundaries", Acta Math. 179, 295-305 (1997).*
- There is a notion of "positive isotropic curvature" which is closely related to the stability of minimal surfaces as well as Ricci flows. Study how this notion comes up and how this is related to the topological sphere theorems. *M. Micallef & J. Moore, "Minimal two-spheres and the topology of manifolds with positive curvature on totally isotropic two-planes", Ann. of Math. 127 (1988), 199-227.*

- Sacks and Uhlenbeck introduced a regularitization technique to the energy functional to develop a general existence theory for minimal spheres in any Riemannian manifolds. Explain this theory. *Ref: J. Sacks & K. Uhlenbeck, "The existence of minimal immersions of 2-spheres", Ann. of Math. 113 (1981), 1-24.*

- Explain how minimal surface theory can be used to study the topology of complete non-compact three-manifold with non-negative Ricci curvature. *Ref: R. Schoen & S.-T. Yau, "Complete three-dimensional manifolds with positive Ricci curvature and scalar curvature", Seminar on Differential Geometry, Ann. of Math. Stud., vol. 102, pp. 209-228, Princeton University Press (1982); G. Liu, "3-manifolds with nonnegative Ricci curvature", Invent. Math 193 (2013), 367-375.*

- There are some interesting characterizations of the catenoid in $\mathbb{R}^3$. Discuss a few of them. *Ref: R. Schoen, "Uniqueness, symmetry, and embeddedness of minimal surfaces", J. Differential Geom. 18 (1983) 791-809; F. Lopez & A. Ros, "On embedded complete minimal surfaces of genus zero", J. Differential Geom. 33 (1991) 293-300.*

Useful Links

- The Not So Short Introduction to LaTeX (https://tobi.oetiker.ch/lshort/lshort.pdf)

- MathSciNet (https://mathscinet.ams.org/mathscinet/)