MATH2058: Analysis I 2021-2022 **Revision Test 1**

Duration: 1.5 hour 14 Oct 2021 8:30 - 10:00

Answer ALL Questions Full Marks: 50

1 (10 marks)**.** Suppose $\lim a_n = 3$. Show by using definitions that

$$\lim_{n\to\infty} \frac{a_n^2+1}{a_n-2} = 10$$

2 (10 marks)**.** Let (x_n) be a sequence. Suppose $\lim(-1)^n x_n = 0$. Is it true that (x_n) converges? Prove your assertion and find the limit **only** if it converges.

3 (10 marks)**.** Let (x_n) be a sequence. Let $a > 0$ be such that $x_1 > \sqrt{a}$. Suppose that (x_n) satisfies the recursive relation

$$x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n})$$

for all $n \geq 1$. Show that (x_n) converges and find its limit.

4 (20 marks)**.** Let (x_n) be a bounded sequence. We call $x \in \mathbb{R}$ a sequential cluster point of (x_n) if for all $\epsilon > 0$ and for all $N \in \mathbb{N}$ there exists $n \geq N$ such that $|x_n - x| < \epsilon$. Define

$$E := \{x \in \mathbb{R} : x \text{ a sequential cluster point of } (x_n)\}$$

i. Show that E is non-empty.

ii. Show that E is a singleton if and only if (x_n) converges.