MATH 2058 - HW 6 - Questions

1 (P.110 Q15)**.** Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) := \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$.

a. Show that f has a limit at $x = 0$.

b. Let $c \neq 0$. Show that f does not have a limit at c using a sequential argument.

2 (P.116 Q4)**.** Prove the following assertions:

a. The limit $\lim_{x\to 0} \cos(1/x)$ does not exist.

b. The limit $\lim_{x\to 0} x\cos(1/x)$ exists and is equal to 0.

3 (P.129 Q10)**.** Show that the absolute function function $f(x) := |x|$ defined on $\mathbb{R}$ is continuous everywhere on $\mathbb{R}$.