

MATH 2058 - HW 1 - Questions

1 (P.44 Q8). Let $X \subset \mathbb{R}$ be a non-empty subset. Let $f, g : X \rightarrow \mathbb{R}$ be functions of bounded ranges.

a. Show that

i. $\sup\{f(x) + g(x) : x \in X\} \leq \sup\{f(x) : x \in X\} + \sup\{g(x) : x \in X\}$

ii. $\inf\{f(x) : x \in X\} + \inf\{g(x) : x \in X\} \leq \inf\{f(x) + g(x) : x \in X\}$

b. Give examples to show that each of the above inequalities can either be strict or equal.

2 (P.44-45 Q11). Let $X, Y \subset \mathbb{R}$ be nonempty subsets. Let $h : X \times Y \rightarrow \mathbb{R}$ be of bounded range. Let $f : X \rightarrow \mathbb{R}$ and $g : Y \rightarrow \mathbb{R}$, defined by

$$f(x) := \sup\{h(x, y) : y \in Y\} \qquad g(y) := \inf\{h(x, y) : x \in X\}$$

for all $x \in X$ and $y \in Y$ respectively. Show that

$$\sup\{g(y) : y \in Y\} \leq \inf\{f(x) : x \in X\}$$