

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MMAT 5120 Topics in Geometry 2021-22**  
**Lecture 5 practice problems**  
**18th February 2022**

- The practice problems are meant as exercise to the students. You are **NOT** required to submit your solutions, but you are encouraged to work through all of them in order to understand the course materials. The problems will be uploaded on Fridays and solutions will be uploaded on Wednesdays before the next lecture.
  - Please send an email to [eclam@math.cuhk.edu.hk](mailto:eclam@math.cuhk.edu.hk) if you have any questions.
1. Let  $z_1, z_2, z_3, z_4$  be four general distinct points, there are  $4! = 24$  many permutations of these four points. Some permutations would give the same value of cross ratio. Can you determine how many distinct values of cross ratio are there in general?
  2. Find a Möbius transformation  $w = f(z)$  that maps  $-1, i, 1$  to  $-1, 0, 1$  respectively.
  3. What is the relation between symmetric points  $z$  and  $z^*$  if we set  $z_3 = \infty$ ?
  4. Do the points  $2 + i, 3, 5, 6 + i$  lie on the same cline?
  5. Find a Möbius transformation that take the circle  $|z| = 4$  to the line  $3x + y = 4$ .