THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT 5120 (2021-22, Term 2)
Topics in Geometry
Homework 3
Due Date: 7th April 2022

1. Let C be a hyperbolic circle. Show that the family of Euclidean circles perpendicular to both C and the unit circle $\partial\mathbb{D}$ is a family of Steiner circles of the first kind with respect to p and $p*$ for some point $p \in \mathbb{D}$. (Here, p and $p*$ are symmetric with respect to the unit circle $\partial\mathbb{D}$.)

2. Show that for any hyperbolic triangle Δpqr, there exists a unique hyperbolic cycle passing through p, q, r.

3. Show that any two horocycles are congruent in hyperbolic geometry.

4. Let C be a hypercycle, and let L be the hyperbolic straight line which shares the same ideal points as C. Show that the perpendicular distance from C to L is the same at every point of C (hence the name **equidistant curve**).

5. Show that the circumference of a hyperbolic circle of hyperbolic radius R is precisely given by $2\pi \sinh R$.

6. (**Pythagorean Theorem**) Let a, b, c be the hyperbolic lengths of the 3 sides of a right-angled hyperbolic triangle, where c is the side opposite to the right angle. Show that $\cosh c = \cosh a \cosh b$.