

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT 5120 (2021-22, Term 2)
Topics in Geometry
Homework 1
Due Date: 3rd March 2022

We always denote by i the imaginary unit $\sqrt{-1}$.

1. Compute the following cross ratios:

- (a) (∞, z_1, z_2, z_3) ,
- (b) (z_0, ∞, z_2, z_3) ,
- (c) (z_0, z_1, ∞, z_3) ,
- (d) (z_0, z_1, z_2, ∞) .

2. Find a Möbius transformation which:

- (a) sends $1 \mapsto 4$, $0 \mapsto i$ and $\infty \mapsto -1$,
- (b) sends $0 \mapsto 0$, $i \mapsto 1$ and $-i \mapsto 2$,
- (c) takes the unit circle $C := \{z \in \mathbb{C} : |z| = 1\}$ to the straight line $x + y = 1$.

3. Find all Möbius transformations which:

- (a) have the fixed points 1 and -1 ,
- (b) have only one fixed point at -1 .

4. Prove that all clines are congruent in Möbius geometry. (*Hint: Apply the Fundamental Theorem of Möbius Geometry*).

5. Let $C := \{z \in \mathbb{C} : |z| = 1\}$ be the unit circle. Find the points z^* symmetric with respect to C for:

- (a) $z = 1$,
- (b) $z = 1/2$,
- (c) $z = i$,
- (d) $z = i/2$,
- (e) $z = 1 + i$,
- (f) $z = (1 + i)/2$.

Try to draw C and all the points z, z^* in the same figure.