MATH 2060B - HW 6
Due Date: 24 March 2021, 23:59

Problems: Ex8.1 P.246: 12, 22, 23 $\hspace{4cm}$ (3 Questions in total)

Textbook: Bartle RG, Sherbert DR(2011). Introduction to Real Analysis, fourth edition, John
Wiley Sons,Inc.

Instruction:

1. Please submit your solution in one pdf file to Blackboard.

2. Rename your file in the form "HW1_ChanTaiMan_1155151031".

3. You are reminded that your HW is graded based on **both** your idea and your presentation

Questions:

1 (P.246 Q12). Let (f_n) be the sequence of functions defined on $\mathbb{R}$ by $f_n(x) := \frac{nx}{1+n^2x^2}$ for all $x \in \mathbb{R}$
and $n \in \mathbb{N}$. Then it is known that (f_n) converges pointwise to 0.

 i. Show that the sequence converges uniformly on $[a, \infty)$ if $a > 0$.

 ii. Show that the sequence does not converge uniformly on $[0, \infty)$

2 (P.246 Q22). Let (f_n) be a sequence of functions defined by $f_n(x) := x + \frac{1}{n}$ for all $x \in \mathbb{R}$ and
$n \in \mathbb{N}$. Define $f(x) := x$ for all $x \in \mathbb{R}$.

 i. Show that (f_n) converges uniformly to f on $\mathbb{R}$.

 ii. Show that (f_n^2) does not converge uniformly on $\mathbb{R}$.

Remark. This shows that uniform convergence may not be preserved by multiplication.

3 (P.246 Q23). Let (f_n), (g_n) be sequences of *bounded* functions on some subset $A \subset \mathbb{R}$ which
converge uniformly on A to f, g respectively.
Show that $(f_n g_n)$ converges uniformly on A to fg.