MATH 2060B - HW 1
Due Date: 25 Jan 2021, 23:59

Problems: Ex6.1 P.171: 4, 10, 13 (3 Questions in total)

Textbook: Bartle RG, Sherbert DR(2011). Introduction to Real Analysis, fourth edition, John Wiley Sons,Inc.

Instruction:

1. Please submit your solution in 1 pdf file to Blackboard.
2. Rename your file in the form "HW1_ChanTaiMan_1155151031".
3. You are reminded that your HW is graded based on **both** your idea and your presentation

Questions:

1 (P.171 Q4)**.** Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} x^2 & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}$.

a. Show that f is differentiable at $x = 0$

b. Find $f'(0)$

2 (P.171 Q10)**.** Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = \begin{cases} x^2 \sin(1/x^2) & x \neq 0 \\ 0 & x = 0 \end{cases}$.

a. Show that g is differentiable for all $x \in \mathbb{R}$.

b. Show that the derivative g' is not bounded on the interval $[-1, 1]$

3 (P.171 Q13)**.** Let $f : \mathbb{R} \to \mathbb{R}$ be a real-valued function and $c \in \mathbb{R}$.

a. Suppose f is differentiable at c. Show that $f'(c) = \lim_{n\to\infty}(n(f(c + 1/n) - f(c)))$

b. Show with an example of f that the existence of sequential limit in part(a) does not imply the existence of $f'(c)$.