

The Chinese University of Hong Kong
Department of Mathematics

MMAT 5140 Probability Theory 2015 - 2016
Suggested Solution to Homework 3

1. P. 151, Q9

(a)

$$\begin{aligned}P(|X| \leq t) &= P(X \leq t) - P(X < -t) \\&= P(X \leq t) - (1 - P(X \geq -t)) \\&= P(X \leq t) + P(X \geq -t) - 1 \\&= 2P(X \leq t) - 1 \\&= 2F(t) - 1\end{aligned}$$

(b)

$$\begin{aligned}P(|X| > t) &= P(X > t) + P(X < -t) \\&= (1 - P(X \leq t)) + (1 - P(X \geq -t)) \\&= 2 - P(X \leq t) - P(X \geq -t) \\&= 2 - 2P(X \leq t) \\&= 2(1 - F(t))\end{aligned}$$

(c)

$$\begin{aligned}P(X = t) &= 1 - P(X > t) - P(X < t) \\&= 1 - (1 - P(X \leq t)) - (1 - P(X \geq t)) \\&= P(X \leq t) + P(X \geq t) - 1 \\&= P(X \leq t) + P(X \leq -t) - 1 \\&= F(t) + F(-t) - 1\end{aligned}$$

2. P. 152, Q11 F is a distribution function. We need to check that

(a) F is increasing and right continuous. This is trivial.

(b) $\lim_{t \rightarrow -\infty} F(t) = 0$ and $\lim_{t \rightarrow \infty} F(t) = 1$. This is also clear.

Hence, F is a distribution function.