The Chinese University of Hong Kong
Department of Mathematics

MMAT 5140 Probability Theory 2015 - 2016
Suggested Solution to Homework 1

1. P. 34, Q4

$$\begin{aligned} P\left(A\bigcap B\right) &= 1 - P\left(\left(A\bigcap B\right)^c\right) \\ &= 1 - P\left(A^c\bigcup B^c\right) \\ &= 1 - \left(P(A^c) + P(B^c) - P\left(A^c\bigcap B^c\right)\right) \\ &= 1 - (0 + 0 - 0) \\ &= 1 \end{aligned}$$

2. P. 34, Q8 We prove it by Mathematical Induction. Let S_n be the statement for $n \geq 1$ and the statement is clearly true for $n = 1$. Suppose S_k is true, then we need to prove that S_{k+1} is true. Since S_k is true, we have $P(\cap_{i=1}^k A_i) = 1$. Setting $A = \cap_{i=1}^k A_i$ and $B = A_{k+1}$ in the last question, we have $P(\cap_{i=1}^{k+1} A_i) = 1$. Hence, S_{k+1} is true and the result follows from the principle of Mathematical Induction.

3. P.35, Q13 We show it by providing a counterexample. Let x be a real number randomly drawn from 0 to 1, that is, for $0 \leq t \leq 1$,

$$P(0 \leq x \leq t) = t.$$

Let E_t be the event that $x \neq t$, then $P(E_t) = 1$ for all $t \in [0, 1]$. To see that, for $\varepsilon > 0$, we have

$$\begin{aligned} P(x \neq t) &\leq P(t < x \leq t + \varepsilon) \\ &= P(0 \leq x \leq t + \varepsilon) - P(0 \leq x \leq t) \\ &= \varepsilon. \end{aligned}$$

Since $\varepsilon > 0$ is arbitrary, we must have $P(E_t) = 1$. However, it is clear that we must have $x \in (0, 1)$ by definition and so

$$P\left(\bigcap_{t\in(0,1)} E_t\right) = P(x \notin (0, 1)) = 0$$