

**THE CHINESE UNIVERSITY OF HONG KONG  
DEPARTMENT OF MATHEMATICS**

MATH3093, Assignment 1

Date Due: Jan 18, 2022

(1) Verify that  $f(x) = e^{inx}$  ( $n \in \mathbb{Z}$ ) is periodic with period  $2\pi$  and that

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{inx} dx = \begin{cases} 1 & \text{if } n = 0, \\ 0 & \text{if } n \neq 0. \end{cases}$$

Using this fact to show that if  $n, m \geq 1$  ( $n, m \in \mathbb{Z}$ ) we have

$$\frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx \cos mx dx = \begin{cases} 0 & \text{if } n \neq m, \\ 1 & \text{if } n = m, \end{cases}$$

and similarly

$$\frac{1}{\pi} \int_{-\pi}^{\pi} \sin nx \sin mx dx = \begin{cases} 0 & \text{if } n \neq m, \\ 1 & \text{if } n = m. \end{cases}$$

Finally, show that

$$\frac{1}{\pi} \int_{-\pi}^{\pi} \sin nx \cos mx dx = 0 \quad \text{for all } n, m \in \mathbb{Z}.$$

(2) Consider the Laplacian  $\Delta u \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  on the disc  $\{(x, y) : x^2 + y^2 < 1\}$ .

(a) Show that in polar coordinates  $U(r, \theta) = u(x, y)$  satisfies

$$(0.1) \quad \frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \theta^2} = 0$$

in  $\{(r, \theta) : 0 \leq r < 1, \theta \in \mathbb{R}\}$ .

(b) Show that all solutions of the form  $F(r)G(\theta)$  to (0.1) are one of the forms

$$r^n(A_n \cos n\theta + B_n \sin n\theta), \quad n = 0, 1, 2, \dots$$