Workshop on Geometric Analysis and General Relativity

The Chinese University of Hong Kong

January 12 - 16, 2018

Title and Abstract

Distinguished Public Lecture

Heat Diffusion in Geometry

Gerhard Huisken
University of Tübingen

Abstract: The lecture will describe how diffusion properties of geometric evolution equations of parabolic type can be used to deform complicated geometric structures into simpler models that can be more easily understood. Specific examples will be the deformation of hyper surfaces by mean curvature flow and the deformation of Riemannian metrics by Ricci flow.

Plenary Talks

Ricci solitons

Huai-dong Cao
Lehigh University

Abstract: In this talk I will discuss some recent progress about the geometry and classifications of Ricci solitons, especially in dimension four.

Some minimal submanifolds generalizing the Clifford torus

Jaigyoung Choe
Korea Institute for Advanced Study

Abstract: The Clifford torus is the simplest non-totally geodesic minimal surface in S^3. It is a product surface, it is helicoidal, and it is a solution obtained by separation of variables. We will see that there are more minimal submaniolfds with these properties in S^n.

A fully non-linear flow with surgery for 2-convex hypersurfaces

Gerhard Huisken
University of Tübingen

Abstract: The lecture describes joint work with Simon Brendle on the deformation of hypersurfaces in a Riemannian ambient manifold, on which the sum of any two principle curvatures is positive everywhere. We prove long-time existence of solutions to a fully non-linear parabolic flow with surgeries that preserves this condition in all ambient spaces satisfying a natural lower bound on its curvature tensor. As a consequence we can prove that such hypersurfaces bound a 1-handle body.

Recent constructions of minimal surfaces and related topics

Nicos Kapouleas
Brown University

Abstract: I will first discuss doubling gluing constructions for minimal surfaces with emphasis on a recent construction for free boundary minimal surfaces in the unit ball (with D. Wiygul: arXiv:1711.00818) and constructions in the round three-sphere using the Linearized Doubling methodology (J. Differential Geom. 106:393-449, 2017); and with P. McGrath: arXiv:1707.08526). I will then discuss desingularization gluing constructions with emphasis on a recent construction for free boundary minimal surfaces in the unit ball (with M. Li: arXiv:1709.08556) and a construction in the round three-sphere (with D. Wiygul: arXiv:1701.05658). Finally I will discuss open problems motivated by the previous constructions and if time permits a recent general construction for Constant Mean Curvature hypersurfaces (with C. Breiner: arXiv:1707.04008) and a construction for ancient solutions of the Ricci flow related to an old question of Page (with S. Brendle: Comm. Pure Appl. Math., 70(7):1366-1401, 2017).

Variational problems for Riemannian functionals and the Generalized Willmore conjecture

Haizhong Li
Tsinghua University

Abstract: In this talk, we present our research works about variational problems of Riemannian functionals on an n-dimensional compact Riemannian manifold (M, g), which include the renormalized volume coefficients functional $\int_M v^{2k}(g)dv_g$, and the Weyl curvature functional $\int_M |W(g)|^{n/2}dv_g$. For a hypersurface in a sphere, we study the generalized Willmore functional and generalized Willmore conjecture. By use of an inequality between the Weyl curvature functional and the generalized Willmore functional, we give some discussions about Generalized Willmore conjecture for 4-dimensional compact hypersurfaces in a sphere.

Invited Talks

The Kähler-Yang-Mills equations

Luis Alvarez-Consul
Instituto de Ciencias Matemáticas

Abstract: We study equations on a principal bundle over a compact complex manifold coupling a connection on the bundle with a Kähler structure on the base. These equations generalize the conditions of constant scalar curvature for a Kähler metric and Hermite-Yang-Mills for a connection. We provide a moment map interpretation of the equations and study obstructions for the existence of solutions, generalizing the Futaki invariant, the Mabuchi K-energy and geodesic stability. We finish by giving some examples of solutions. This lecture is based on joint work with M. Garcia-Fernandez and O. Garcia-Prada. Subsequent applications to the study of gravitating vortices and cosmic strings will be discussed in O. Garcia-Prada's lecture.

Gravitating vortices and the Einstein-Bogomol'nyi equations

Oscar Garcia-Prada
Instituto de Ciencias Matemáticas

Abstract: After briefly recalling the Khler-Yang-Mills equations on a holomorphic bundle over a compact complex manifold (see L. Alvarez-Consul's lecture for more details), we study the existence of solutions to these equations by applying dimensional reduction methods to the product of the complex projective line with a compact Riemann surface. The resulting equations, that we call gravitating vortex equations, describe abelian vortices on the Riemann surface coupled to a metric. They generalize the Einstein-Bogomol'nyi equations studied in physics in connection with cosmic strings (based on joint work with L. Alvarez-Consul, M. Garcia-Fernandez and V. Pingali).

TBA

Karsten Gimre
Harvard University

Abstract: TBA

Automorphisms of a symmetric product of a curve

Tomas Gomez
Instituto de Ciencias Matemáticas

Abstract: We show that all the automorphisms of the d-fold symmetric product of a smooth projective curve of genus $g > 2$, with the condition $d > 2g - 2$, are induced by automorphisms of the curve. This is joint work with Indranil Biswas.

Asymptotic behavior of the inverse mean curvature flows in the hyperbolic space

Pei-Ken Hung
Columbia University

Abstract: Inverse mean curvature flow is an expanding curvature flow which has many applications in mathematical relativity such as the Riemanninan Penrose Ineqaulity proved by Huisken and Ilmanen. In many situations it is important to understand the limiting geometry of the flow. I will discuss the asymptotic behavior in the case the ambient manifold is the hyperbolic space. It is joint work with Mu-Tao Wang.

Linear Stability of Higher Dimensional Schwarzschild Black Holes

Jordan Keller
Black Hole Initiative, Harvard University

Abstract: The Schwarzschild-Tangherlini black holes are higher-dimensional generalizations of the Schwarzschild spacetimes, comprising a static, spherically symmetric family of black hole solutions to higher-dimensional vacuum gravity. The physical relevance of such solutions is intimately related to their stability under gravitational perturbations. This talk will address results on the linear stability of the Schwarzschild-Tangherlini black holes, part of ongoing joint work with Pei-Ken Hung and Mu-Tao Wang.

Complexifications in minimal surface theory

Hojoo Lee
Korea Institute for Advanced Study

Abstract: Many beautiful results for minimal hypersurfaces fail utterly in higher codimensions, as well-known in Lawson-Osserman's 1977 paper *Non-existence, non-uniqueness and irregularity of solutions to the minimal surface system*. To illustrate that Euclidean four-space is a wonderland, we shall sketch various generalizations of Bernstein Theorem, and present explicit examples of minimal varieties.

Boundary effect of scalar curvature

Pengzi Miao
University of Miami

Abstract: We consider the implications of nonnegative scalar curvature of a compact 3-manifold on the geometry of the manifold boundary. Such considerations are prompted by the quasi-local mass problem in general relativity. When the manifold boundary has positive Gauss curvature, fundamental results were given by Shi and Tam in 2002 via applications of the positive mass theorem. In this talk, we discuss some recent work that is motivated by Shi-Tam's theorem. The talk will be based on joint work with Christos Mantoulidis, and with Siyuan Lu, respectively.

Variations on the theme of volume

Tommaso Pacini
University of Torino

Abstract: The standard volume functional can be defined for any submanifold: the "flip side" of this generality is that it generally does not capture any special geometric features of the submanifold. Lagrangian submanifolds are an important exception: in this case the volume functional has several important properties related to ambient curvature, maslov indices, etc. Joint work with J Lotay (UCL) has showed that, by adding an appropriate weight function, we can extend this theory from the symplectic to the complex category, also obtaining several new features. The seminar will provide a broad survey of these results.

A strong stability condition on minimal submanifolds

Chung-Jun Tsai
National Taiwan University

Abstract: It is well known that the distance function to a totally geodesic submanifold of a negatively curved ambient manifold is a convex function. We identify a strong stability condition on minimal submanifolds that generalizes the above scenario. In particular, if a closed minimal submanifold Σ is strongly stable, then:

1. The distance function to Σ satisfies a convex property in a neighborhood of Σ, which implies that Σ is the unique closed minimal submanifold in this neighborhood, up to a dimensional constraint.

2. The mean curvature flow that starts with a closed submanifold in a C^1 neighborhood of Σ converges smoothly to Σ.

Many examples, including several well-known types of calibrated submanifolds, are shown to satisfy this strong stability condition. This is based on joint work with Mu-Tao Wang.

Limit of Wang-Yau quasi-local mass of unit spheres at spatial infinity

Ye-Kai Wang
National Cheng Kung University

Abstract: Wang-Yau quasi-local mass was proposed by Professor Mu-Tao Wang and Shing-Tung Yau in 2008. It satisfies a positive mass theorem and is zero for surfaces in flat spacetime. It is also consistent with the classical limits. We consider unit spheres in an asymptotically initial data set and try to evaluate their quasi-local mass as the center approaches infinity. This is a joint work with Po-Ning Chen, Mu-Tao Wang, and Shing-Tung Yau.

Harmonic Maps and the Einstein Equation:
constructing a new set of stationary solutions in 5 dimension

Sumio Yamada
Gakushuin University

Abstract: We introduce a method of constructing 5 dimensional stationary spacetimes which are solutions to the vacuum Einstein equation. The method is well-established in 4 dimension, sometimes called Ernst formalism, but in 5 dimension there are new set of difficulties, including the formulation of boundary value data, and the existence of harmonic maps into rank 2 symmetric spaces. This is a joint work with Marcus Khuri and Gilbert Weinstein.