2022-23 MATH2048: Honours Linear Algebra II Homework 2

Due: 2022-09-23 (Friday) 23:59

For the following homework questions, please give reasons in your solutions. Scan your solutions and submit it via the Blackboard system before due date.

1. Please prove that the addition and scalar multiplication operations of quotient space are well defined. i.e. suppose $\vec{v} + W = \vec{v}' + W$, then for any $\vec{v}'' + W \in V/W$, the following equations hold.

   (a) $(\vec{v} + W) + (\vec{v}'' + W) = (\vec{v}' + W) + (\vec{v}'' + W)$

   (b) $a \cdot (\vec{v} + W) = a \cdot (\vec{v}' + W)$ for any $a \in F$

2. With the addition and scalar multiplication of quotient space defined in lecture note, show that V/W is a vector space over F.

3. Sec. 1.7: Q7

4. (Extension to Sec. 2.1: Q18) Please find **ALL** linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $N(T) = R(T)$.

5. Sec. 2.1: Q25(c)

6. Sec. 2.1: Q26

The following are extra recommended exercises not included in homework.

1. Sec. 1.7: Q6

2. Sec. 2.1: Q14

3. Sec. 2.1: Q17

4. Sec. 2.1: Q24(b)