2022-23 MATH2048: Honours Linear Algebra II Homework 1

Due: 2022-09-16 (Friday) 23:59

For the following homework questions, please give reasons in your solutions. Scan your solutions and submit it via the Blackboard system before due date.

1. Let $n \in \mathbb{N}^+$ and $\mathbb{F}_2 = \{0, 1\}$ be the binary field, i.e. the finite field of order 2.
   (a) Prove that $M_{n\times n}(\mathbb{F}_2)$, the collection of all $n \times n$ matrices with entries in $\mathbb{F}_2$, is a vector space.
   (b) Prove or disprove that $S = \{A \in M_{n\times n}(\mathbb{F}_2) : \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} = 0\}$ is a subspace of $M_{n\times n}(\mathbb{F}_2)$.
2. Suppose W_1 and W_2 are two subspaces of V, please give a necessary and sufficient condition such that $W_1 \cup W_2$ is a subspace of V and prove it.
3. Textbook (Friedberg). Sec. 1.3: Q26
4. Textbook (Friedberg). Sec. 1.6: Q31
5. Textbook (Friedberg). Sec. 1.6: Q33(b)

The following are extra recommended exercises not included in homework.

1. Textbook (Friedberg). Sec. 1.3: Q31
2. Textbook (Friedberg). Sec. 1.4: Q15
3. Textbook (Friedberg). Sec. 1.5: Q15
4. Textbook (Friedberg). Sec. 1.6: Q29