THE CHINESE UNIVERSITY OF HONG KONG

Department of Mathematics

MATH4010 Functional Analysis 2022-23 Term 1

Homework 3

Deadline: 2022-10-03 Monday

Notice:

- All the assignments must be submitted before the deadline.
- Each assignment should include your name and student ID number.

1. Show that $p(x) = \limsup x(n)$, where $x = (x(n)) \in \ell_\infty$, $x(n) \in \mathbb{R}$, defines a sublinear functional on ℓ_∞.

2. Let p be the Minkowski functional for an open convex neighborhood U of 0 in a normed space X.

   (a) Show that for $x \neq 0$, $p(x) = 0$ if and only if $x \in tU$ for every $t > 0$, so U is "unbounded in the direction of the vector x."

   (b) Show that $p(x) \leq 1$ if $x \in U$, and $p(x) \geq 1$ if $x \notin U$.

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