THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2058 Honours Mathematical Analysis I 2022-23
Tutorial 6
20th October 2022

- Tutorial problems will be posted every Wednesday, provided there is a tutorial class on the Thursday same week. You are advised to try out the problems before attending tutorial classes, where the questions will be discussed.
- Solutions to tutorial problems will be posted after tutorial classes.
- If you have any questions, please contact Eddie Lam via echlam@math.cuhk.edu.hk or in person during office hours.

1. Determine whether the following sequences are Cauchy or not from definition.
   (a) $x_n = \frac{n^2-1}{n^2+3}$.
   (b) $x_n = 1 - \frac{1}{2!} + ... + \frac{(-1)^{n+1}}{n!}$
   (c) $x_1, x_2 \in \mathbb{R}$, and define inductively $x_n = \frac{1}{2}(x_{n-1} + x_{n-2})$.
2. Suppose that $\{x_n\}$ is a sequence satisfying the property where $\lim |x_{n+1} - x_n| = 0$, is x_n necessarily Cauchy?
3. Suppose that $\{x_n\}$ is a sequence satisfying that there is some constant $1 > C > 0$ so that
$$|x_{n+1} - x_n| \leq C|x_n - x_{n-1}|.$$
   Show that $\{x_n\}$ is Cauchy.
4. Show that a montone sequence that contains a Cauchy subsequence is itself Cauchy.
5. Show directly that every bounded and monotone sequence is Cauchy, without using monotone convergence theorem (i.e. pretend that you don't know the convergence of the sequence).
6. In class, we have deduced the Archimedean property (AP) and Cauchy's theorem as consequences of the axiom of completeness. Prove the converse: AP + Cauchy's theorem implies completeness.