A Modified Potential Field Model for Shape Interpolation

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Abstract
Geometric properties of a two-dimensional polygonal region are inherently useful in shape interpolation. Traditionally, shape interpolation begins with a collection of characteristic points that define the original polygons. The collection of characteristic points is confining in nature and does not deliver the desired solution at all times. This paper details an interpolation technique that takes a wholistic approach. The model draws on the principles and physical properties of potential fields. The model presented is adaptable to the three-dimensional space with little change in the computational complexity.

I. INTRODUCTION

Geometric interpolation is an important research domain within the fields of computer graphics, pattern recognition, rapid prototyping and cartographic visualization. The most common approach derives intermediate scenes using geometric data from a pair of leading and terminal images. Typically, a set of characteristic points on the border of two-dimensional polygonal regions of the leading image are identified and matched against those of the terminal image [1, 2]. The critical issue in such a methodology is to determine a suitable set and number of corresponding points on both images [3]. The approach tends to fall apart when the leading and terminal images differ substantially in size, shape, and complexity.

This paper proposes a wholistic method to solving the problem of shape interpolation between two polygonal regions. The argument stems from the fact that all intermediate scenes would be some functions of the pair of parent (i.e. leading and terminal) images [4]. The conventional approach that considers geometric description of a polygon in terms of some characteristic points is discontinuous in nature and the shape interpolation thereof would not have observed the subtle difference in feature characteristics of a polygon. It is suggested in this paper that the functional relations of intermediate scenes with the parent images would behave as a form of potential field. This is to say that the shape of the parent polygons would affect that of the intermediate polygons and the degree of influence is inversely proportional to distances from the parent polygons.

II. CONVENTIONAL APPROACHES TO SHAPE INTERPOLATION AND THEIR SHORTCOMINGS

There are two broad classes of algorithmic approaches to geometric interpolation: linear and terrace. Linear interpolation has been known to yield criss-crossing boundaries [5,6]. Sederberg, et.al. [7] proposed the use of angles and lengths as additional measures to controlling criss-crossing boundaries. Shapira and Rappoport [1] designed the star-skeleton mechanism to tackle criss-crossing boundaries. The terrace approach considers the leading and terminal images as two horizontal planes or terraces stacked at different heights [2,8]. Characteristic points on the outer border of polygonal regions of each terrace form a closed contour. These contours at different heights define a three-dimensional space whereby intermediate planes at various heights are derivable by slicing the three-dimensional object bound by the two terraces at both ends.

It is apparent from the above discussion that the conventional means of interpolation hinges on the geometric shapes of two-dimensional polygonal regions. The procedure requires a generalized description of the polygonal regions (as in defining their shapes with characteristic points) and then phases in localized processing to obtain characteristic points for the intermediate scenes. The generalized description keeps the number of points to the minimum while localized processing computes the solution one point or one segment at a time. The geometric approach does result in computational efficiency. But the geometric

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assumption lends severe restriction on the graphics appearance of the interpolated result.

Let the two parent polygons be called polygon I and polygon II. The leading polygon I is defined as $P(p_i(x_i,y_i) \in P, i=1,2,3,....,n)$ and the terminal polygon II as $Q(q_i(x_i,y_i) \in Q, i=1,2,3,....,m)$. There exists also a series of intermediate polygons T's between P and Q such that $T(t_i(x_i,y_i) \in T, i=1,2,3,....,u)$. Equation (1) describes the relationship between T and the parent polygons P and Q. Equation (1) also implies that the intermediate polygons T are some functions of the parent polygons.

$$T = WP + W'Q \quad (1)$$

where W and W' are collective weights (expressed in terms of inverse distance square or similar functions) of T with respect to parent polygons P and Q.

Equation (2) expresses the same relationship in the plane coordinate system. Equation (2) indicates that every point on the interpolated polygons hinges on points on the parent polygons. Indeed, the shape of the parent polygons and their inter-relationship govern the appearance of the intermediate polygons.

$$x_k^T = \sum_{i=1}^{n}(w_{kixx}^{TP}x_i^P + w_{kixy}^{TP}y_i^P) + \sum_{j=1}^{m}(w_{kjxx}^{TQ}x_j^Q + w_{kjxy}^{TQ}y_j^Q)$$

$$y_k^T = \sum_{i=1}^{n}(w_{kiyx}^{TP}x_i^P + w_{kiyy}^{TP}y_i^P) + \sum_{j=1}^{m}(w_{kjyx}^{TQ}x_j^Q + w_{kjyy}^{TQ}y_j^Q) \quad (2)$$

where:

- (x, y) are coordinate pairs of points defining polygons P,Q, and T
  {P ≡the leading polygon; Q ≡the terminal polygon; and T ≡the intermediate polygons}
  $\{i=1,2,....,n; j=1,2,....,m; k=1,2,....,u \mid i, j, k \in$ Integer};
- n ≡maximum number of points defining polygon P;
- m ≡maximum number of points defining polygon Q;
- u ≡maximum number of points defining polygon T; and
- w ≡coefficients between characteristic points of T and their corresponding points on the parent polygons P and Q.

Characteristic points that summarize the geometric shapes of the parent polygons form the bases for deriving intermediate polygons. While generalization as such increases computational efficiency, there are fundamental problems with this approach. First of all, characteristic points capture the overall essence of a polygon but fail to record the minute details. This would influence the geometric association of the two parent polygons thereby introducing disagreement into the interpolation procedure. The solutions thus obtained will carry some degree of errors. Furthermore, partial processing of characteristic points along the polygon boundaries is not a total approach. The quality of derived solutions is questionable.

III. A WHOLISTIC APPROACH TO SHAPE INTERPOLATION

Other than geometric relationships (such as positions, angles, and distances), two polygons can be related on the basis of Newtonian Potential. The argument behind the approach is that potential fields are governed by mathematical imperatives [9]. Here, we assume that the boundary of a polygonal region carries an electrical charge. The region within the polygon would be a charged field, referred by some as electrostatic, electromagnetic, or potential fields. Some researchers applied the said principle to derive the skeleton line of a single polygonal region with measurable degrees of success [10,11]. Others demonstrated the same in their search for characteristic points that describe a polygonal region [12].

We postulate that the potential field for two interacting polygonal regions (whether partial, total, or non-overlap) can be modeled. We extend the argument further to say that intermediate polygons between the two parent polygons can be derived from their potential field relations. The motivation for the approach presented in this paper is to provide a wholistic means to shape interpolation. By considering the physical phenomenon of potential fields, a polygon must be considered in totality. The fact that potential fields of a continuous nature relate the two parent polygons also infers that the intermediate polygons will be shaped in accordance with the parent polygons.

Equation (2) illustrates that shape interpolation is a function of all the points on the borders of the parent polygon. Numerical computation of Equation (2) becomes feasible only when coefficients w's are known. This requirement is difficult, if not impossible, to fulfill. Although it is not possible to know the shape of the intermediate polygons, the basic qualities of these polygons are known. For example, an intermediate polygon expects to resemble more closely to the leading polygon I at the beginning of the transition to polygon II. An intermediate polygon in the latter part of the transition process will resemble more of the terminal polygon II. It can thus be said that the shape of the intermediate polygons would vary in accordance with the parent polygons such that the one in closer proximity would exert a greater degree of influence. The potential field approach exhibits such a force field and expects to improve the graphics appearance of the intermediate polygons. Furthermore, the wholistic approach expects to eradicate the inherent deficiency of the geometric approach that gives rise to criss-crossing lines.

The potential field method to shape interpolation

Newtonian Physics states that the potential field intensity V to a border point is inversely proportional to the distance r from the border point [10] as illustrated in Equation (3) below. The fact that the potential field intensity and its distributional pattern vary inversely with distance from the border point also indicates that potential field is a function of the geometric shape of the polygon. It follows that potential field intensity can serve as a controlling parameter to model changes in geometric shapes.

$$V = \frac{k}{r} \qquad (3)$$

where V is the potential field intensity; k is a constant; and $r = ((x - x_0)^2 + (y - y_0)^2)^{\frac{1}{2}}$ is distance of the potential field point (x, y) from the border point (x_0, y_0).

We suggest that the two parent polygons be viewed as two electrical sources or electrodes of opposite polarity. Equation (3) reveals that the potential field between the charged sources would display a pattern of monotony, i.e., the lines of equal potential field intensity would form closed polygons and not intersect each other. Furthermore, the shape of these intermediate polygons would vary disproportionately and in consideration of their respective distances from the source polygons.

The model development

Figure 1 is a graphic illustration of the potential field of Equation (3). It is apparent from Figure 1 that the potential field intensity varies disproportionately with distance. The rate of change is greater when the distance from the electrical source is smaller (i.e., $r < r_0$). On the contrary, the rate of change dissipates as the distance increases (i.e., $r > 1/r_0$). This pattern of change corresponds with our earlier statement that points in closer proximity to the source would be affected more than those at some distance away. The corollary is that the shape of the intermediate polygons of potential field intensity would take a greater resemblance to the parent polygons when the distance apart is small. It is also evident from Figure 1 that the modeling strength crumbles very quickly with increasing distances from parent polygons at both ends.

$$V = \frac{k}{u^r} \qquad (4)$$

where $\{1 < u < \sqrt[r]{r} \mid u \in \text{Real}\}$; and V, k, r as defined in Equation (3).

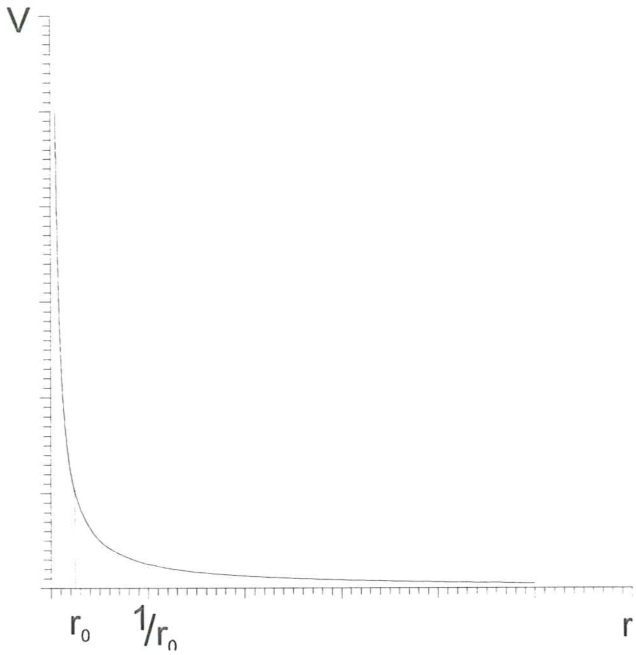

Figure 1. Potential field intensity versus distance

Equation (4) is an attempt to weigh down the abrupt distance effect of the original potential field model by rendering a smoother progression of intermediate polygons between the parent polygons at both ends. The diffused distance effect is reflected in Figure 2. The rate of change (viz. the shape effects of the intermediate polygons) gradually levels off when the value of u approaches 1. Conversely, the rate of change approximates that of the original potential field model

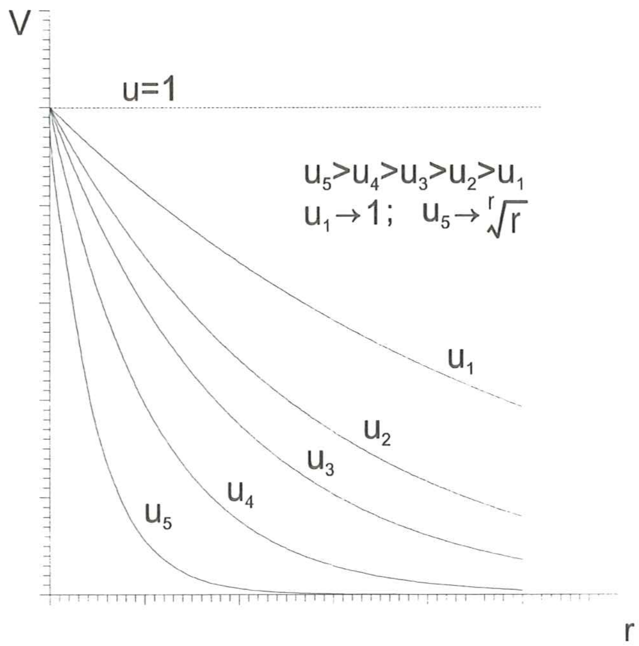

Figure 2. Potential field intensity versus modified distance

when u approaches $\sqrt[u]{r}$, which varies with the maximum distance between boundaries of the parent polygons. Because $\sqrt[u]{r}$ is a variable, it is therefore not possible to suggest a suitable value for u to regulate the change effects of the potential field intensity with distance. A suitable u can be chosen through visual examination of the modified potential field to assess the desired degree of smoothness (an example is provided in Figure 8). Indeed, equation (4) is adaptable to all situations and the potential field intensity can be adjusted in accordance with shape variations and sizes of the source polygons.

Technical implementation of the modified potential field approach

We know from Equation (2) that every point on the borders of the parent polygons do influence the interpolation of intermediate polygons. The inherent weakness of the conventional method that describes a border with characteristic points must thus give way to a better approach. We suggest the rasterized method [11,13] to describing a border for reasons of impartiality and wholeness. It is also evident from Figure 3 that the rasterized coordinates form a tight and objective description of the charged field.

Consider that $I(I_i(x_i, y_i) \in P \mid i=1,2,....n')$ and $II(II_i(x_i,y_i) \in Q \mid i=1,2,....m')$ are respectively rasterized coordinates for the parent polygons P and Q. Then the potential field between the charged polygons I and II is expressed in Equation (5).

$$V = -\sum_{i=1}^{n'} \frac{k_I}{u^{r_i^I}} + \sum_{i=1}^{m'} \frac{k_{II}}{u^{r_i^{II}}} \qquad (5)$$

where :

- V is the potential field intensity;
- k_I and k_{II} are respectively constants for polygons I and II;
- u is the smoothness factor for the potential field;
- r^I and r^{II} are respectively distances from rasterized points on the borders of polygons P and QI;
- n' is the maximum number of rasterized coordinates for polygon P; and
- m' is the maximum number of rasterized coordinates for polygon Q.

An interpolation routine

Equation (5) can derive potential field intensity on the basis of two numerical models: grid versus tessellation. The grid model assumes a uniform structure with regular spacing (Figure 4). The tessellation model defines a triangulated irregular network of known points (Figure 5). Both models interpolate the potential field intensity for any point t(x,y) from a set of known intensity values.

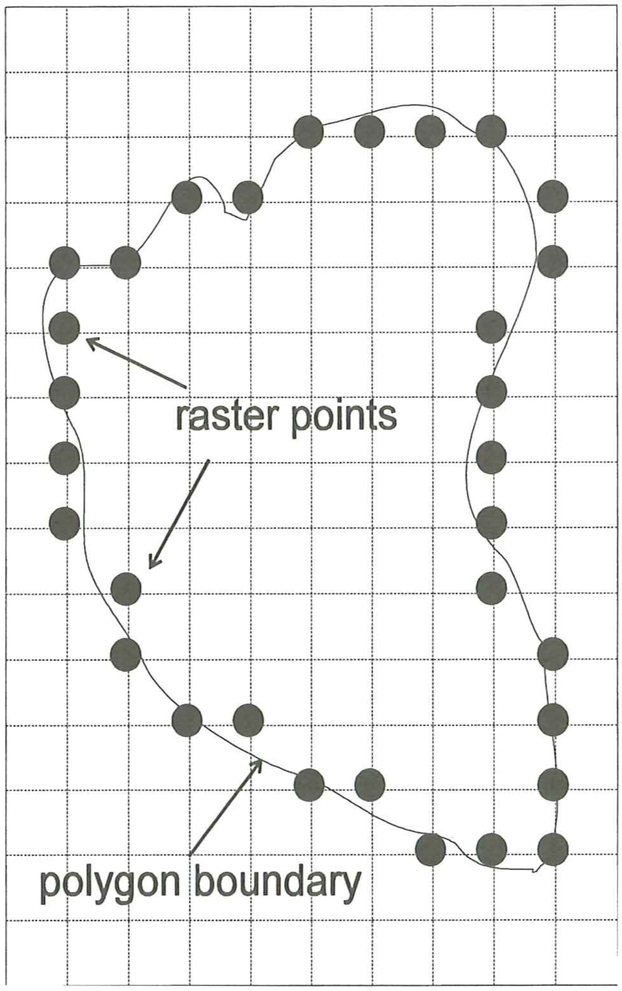

Figure 3. A rasterized approach to defining polygon boundary (adapted from [11])

Equation (6) lists the grid-based numerical model for interpolating an unknown potential field intensity from four points with known intensity values.

$$V = \sum_{i=1}^{4} d_i V_i / \sum_{i=1}^{4} V_i \qquad (6)$$

where V is the potential field intensity; and d_i is distance to known intensity values V_i .

Conversely, Equation (7) shows the tessellation-based numerical model for interpolating an unknown value of potential field intensity from three vertices with known intensity values.

$$\begin{vmatrix} x & y & v & 1 \\ x_1 & y_1 & v_1 & 1 \\ x_2 & y_2 & v_2 & 1 \\ x_3 & y_3 & v_3 & 1 \end{vmatrix} = 0 \qquad (7)$$

where V is the potential field intensity; and $x_i\, y_i\, v_i$ are vertices with known intensity values

Having computed a raster field of potential field in-

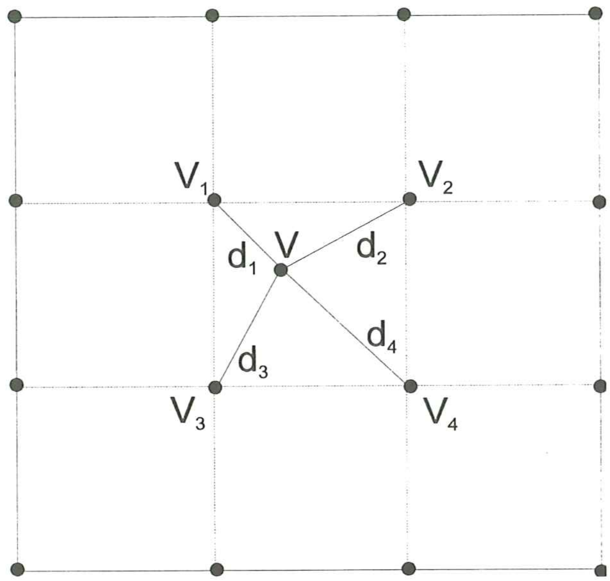

Figure 4. A grid-based numerical model

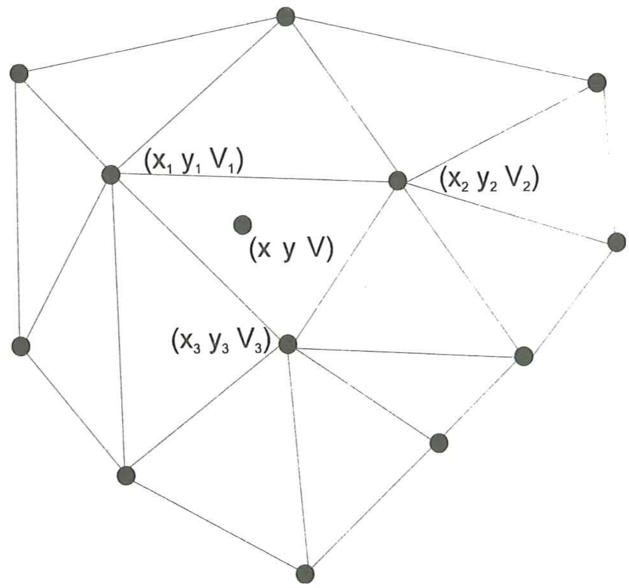

Figure 5. A tessellation-based numerical model

tensity, various contour interpolation routines are available [14] to draw contour lines of equal potential field intensity. These contour lines are, in fact, interpolated boundaries of the intermediate polygons.

IV. AN EMPIRICAL TEST

We tested our algorithm with an empirical example containing two parent polygons (Figure 6). In this case, Polygon I contains a series of concave and convex corners and is uniquely different from Polygon II that is triangular in shape. A potential field between the two parent polygons, as derived from Equation (3), is shown in Figure 7. It is evident from the figure that the intermediate polygons assume the shape of either Polygon I or Polygon II as prescribed by the distances away from each.

Figure 8 shows a modified potential field based on Equation (4) in which u was assigned an arbitrary value of 1.2. A u value approaching 1 is selected to exert a greater amount of smoothing of the original potential field model (see also Figure 2). It is distinctively clear from Figure 8 that the modified potential field yields a more gradual transition of shape changes between the parent polygons I and II. A contour plot of Figure 8 shows the intermediate polygons at various intervals (Figure 9). The intermediate polygons are non-intersecting in nature and their gradual change in shape and complexity is reflective of the combined influence of the parent polygons.

V. CONCLUSION AND RECOMMENDATION

While an empirical example may not be conclusive, we have demonstrated indeed that a wholistic approach to shape interpolation is possible with the potential field model. Our empirical result is rather encouraging. We have shown that intermediate polygons can be extracted effectively and with due regard to the shapes of the parent polygons. The potential field approach seems to resolve the problem of intersecting contours on the one hand and maintain geometric inheritance of the parent polygons on the other. But more importantly, it overcomes the correspondence problem of mapping vertices and edges of the parent polygons. Our method employs a rasterized approach to describing the boundary of a polygonal region for a more compact and complete representation. The raster representation also renders computational efficiency in the algorithmic development.

The modified potential field model was derived with a curve adjusting function that approximates the behavioral pattern of a potential field. The parameter for adjustment can only be determined visually at this stage. While the adjustment may not be impeccable, the function serves adequately as a smoothing mechanism that remains continuous in nature with definable lower and upper bounds for its parameters.

We see that our method can be used in the animation of spatial and geographic events over time [15]. We intend to define further a stepping function for the intermediate polygons to facilitate time series animation. The desired function will correlate potential field

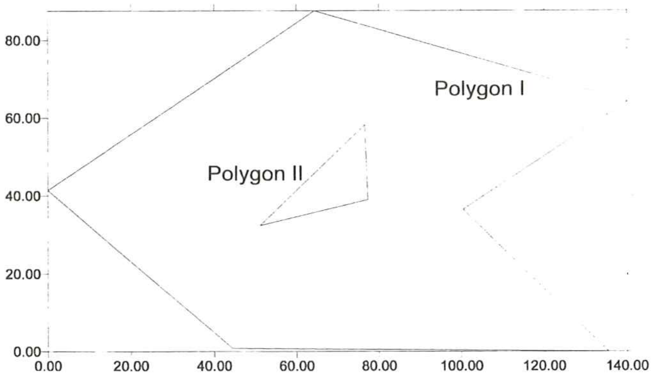

Figure 6. An empirical example containing two parent polygons

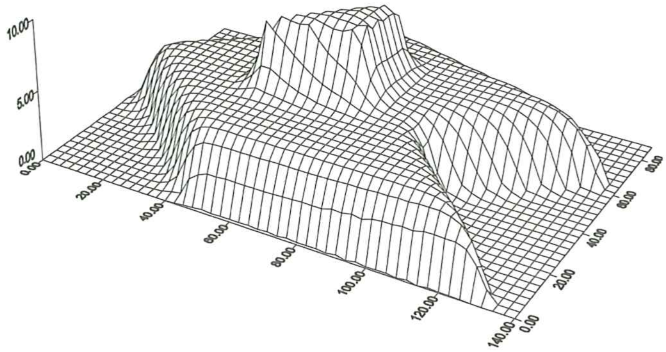

Figure 7. A potential field based on Equation (3)

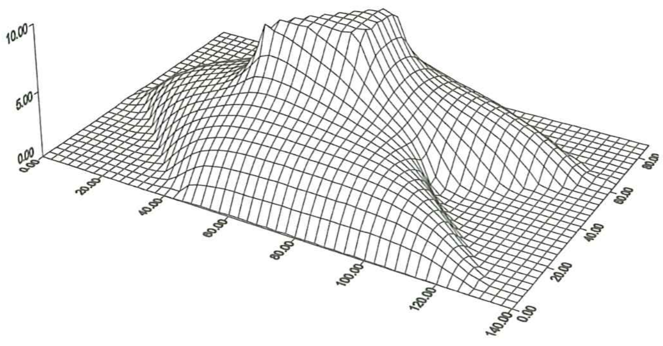

Figure 8. A modified potential field based on Equation (4)

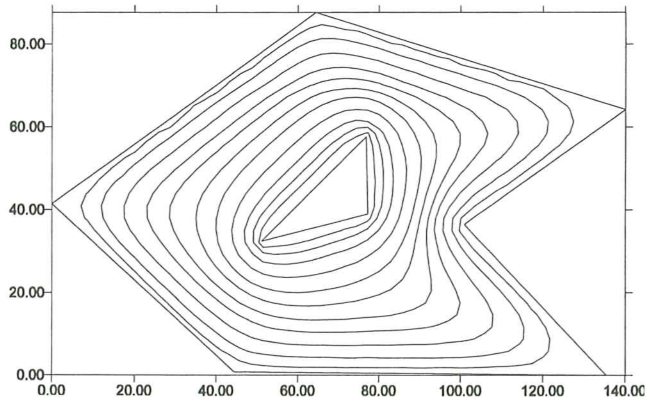

Figure 9. The derivation of intermediate polygons from the modified potential field in Figure 8

intensity with time such that direct inference of a time slice between a pair of images would become amenable. Although not demonstrated in this paper, the method can be augmented to handle intermediate changes in three-dimensional space [16]. Our method expects to play a role in the rapid prototyping technology which has been designed to directly produce functional prototype parts and shape-conformable embedded electronic structures from computer aided design (CAD) models. To manufacture a part, the CAD model is decomposed into slices of objects of arbitrary geometry or shapes. Our method would yield slices of geometrically continuos shapes to be manufactured in alternating deposition and shaping steps.

ACKNOWLEDGMENT

This work reflects partial research findings of the Project *HKU 172/95H* funded by the Hong Kong Research Grants Council Competitive Earmarked Research Grant.

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Sponsored by Ministry of Education, Ministry of Science and Technology, National Center for Remote sensing, National Natural Science Foundation of China, Nanjing University, Beijing Normal University and The Association of Chinese Professionals in GIS, an international forum on advanced remote sensing science has been held at Nanjing University between June 7-15, 1999. The forum invited 13 remote sensing scientists working in the US, Canada and France and attracted 187 attendants from 57 organizations in China. The lecturers included Drs. Shunlin Liang, Xiaowen Li, Xiaohai Yan, Zhanqing Li, Jingming Chen, Zhaoliang Li, Guoqing Sun, Jiancheng Shi, Yongkang Xue, Xu Liang, Yong Wang, Limin Yang, and Peng Gong. Professor Guanhua Xu, Vice Minister of Ministry of Science and Technology, Professor Shu Sun, Deputy Director General of the National Natural Science Foundation of China, Professor Shusheng Jiang, President of Nanjing University, and Academicians Shupeng Chen, Deren Li, Qingxi Tong and Jianmin Xu, and Professor Huadong Guo, Director of the Institute of Remote sensing Application, Chinese Academy of Sciences attended this forum.