# MATH1010 University Mathematics 2015-2016
## Assignment 3
## Due: 26 Oct 2013 (Monday)

Answer all questions.

1. Let $f : [a, b] \to \mathbb{R}$ be a function which is continuous on $[a, b]$ and differentiable on $(a, b)$. Suppose $f'(x) = 0$ for any $x \in (a, b)$. Prove that $f$ is a constant function.

2. Using the mean value theorem to prove for $0 < y < x$ and $p > 1$,

$$py^{p-1}(x - y) < x^p - y^p < px^{p-1}(x - y).$$

3. Using the mean value theorem to prove that for $0 \leq x_1 < x_2 < x_3 \leq \pi$,

$$\frac{\sin x_2 - \sin x_1}{x_2 - x_1} > \frac{\sin x_3 - \sin x_2}{x_3 - x_2}.$$

4. Using the mean value theorem to prove that for $x > 0$,

$$\frac{x}{1 + x} < \ln(1 + x) < x.$$

   Hence, deduce that for $x > 0$,

$$\frac{1}{1 + x} < \ln\left(1 + \frac{1}{x}\right) < \frac{1}{x}.$$

5. By applying the mean value theorem, prove that the equation

$$a_1 x + a_2 x^2 + \cdots + a_n x^n = \frac{a_1}{2} + \frac{a_2}{3} + \cdots + \frac{a_n}{n + 1}$$

   has a root between 0 and 1.

6. Let $f(x)$ be a continuous function defined on $[0, \infty)$ such that
   - $f(0) = 0$,
   - $f'(x)$ exists and is monotonic increasing on $(0, \infty)$.

   Prove that

$$f(a + b) \geq f(a) + f(b)$$

   for $0 \leq a \leq b \leq a + b$.

7. Let $A$ be a subset of $\mathbb{R}$ and $f : A \to \mathbb{R}$ be a function. Suppose that there exists $L > 0$ such that

$$|f(x) - f(y)| \leq L|x - y|$$

   for any $x, y \in A$, then $f$ is said to be satisfying the **Lipschitz condition** on $A$.

   Prove that $\sin x$ satisfies the Lipschitz condition on $\mathbb{R}$.

**End**