

MATH1010 University Mathematics 2014-2015
Assignment 5
Due: 28 Nov 2014 (Friday)

Answer all questions. **Last modified: 19 Nov 2014**

1. Evaluate the following indefinite integrals.

(a) $\int x^2(5-x)^4 dx$

(e) $\int (2x-3)^{10} dx$

(b) $\int \frac{(\sqrt{2x} - \sqrt[3]{3x})^2}{x} dx$

(f) $\int \frac{x dx}{(x^2-1)^{\frac{3}{2}}}$

(c) $\int \tan^2 x dx$

(g) $\int \frac{1}{x^2} \sin \frac{1}{x} dx$

(d) $\int \frac{e^{3x} + 1}{e^x + 1} dx$

(h) $\int \frac{dx}{\sqrt{1+e^{2x}}}$

2. Evaluate the following indefinite integrals (here, $a > 0$ always denote a positive constant).

(a) $\int \frac{dx}{\sqrt{x}(1+x)}$

(d) $\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

(b) $\int \frac{x^2 + 1}{x^4 + 1} dx$

(e) $\int \sqrt{a^2 - x^2} dx$

(c) $\int \frac{dx}{x \ln x \ln(\ln x)}$

(f) $\int \sqrt{\frac{x-a}{x+a}} dx$

3. Evaluate the following indefinite integrals.

(a) $\int x^2 \ln \frac{1-x}{1+x} dx$

(c) $\int x \tan^{-1} x dx$

(b) $\int x^3 \sin 2x dx$

(d) $\int x^3 e^{-x^2} dx$

4. Evaluate the following indefinite integrals.

(a) $\int \frac{2x+3}{(x-2)(x+5)} dx$

(d) $\int \frac{dx}{x^4-1}$

(b) $\int \frac{x^3+1}{x^3-5x^2+6x} dx$

(e) $\int \frac{dx}{1+\sqrt{x}}$

(c) $\int \frac{dx}{(x+1)(x+2)^2(x+3)^3}$

(f) $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx$

5. Evaluate the following indefinite integrals.

(a) $\int \cos^5 x dx$

(c) $\int \frac{1}{\sin^3 x \cos^5 x} dx$

(b) $\int \sin^2 x \cos^4 x dx$

(d) $\int \sin 5x \cos x dx$

6. (a) Show that the integral

$$I_n = \int \frac{dx}{\sin^n x}, \quad n > 2,$$

satisfies the reduction formula

$$I_n = -\frac{\cos x}{(n-1)\sin^{n-1}x} + \frac{n-2}{n-1}I_{n-2}.$$

- (b) Use the reduction formula above to calculate $\int \frac{dx}{\sin^5 x}$.

7. Evaluate the following definite integrals.

(a) $\int_{-1/2}^{1/2} \frac{dx}{\sqrt{1-x^2}}$

(f) $\int_{-1}^1 \frac{xdx}{\sqrt{5-4x}}$

(b) $\int_0^2 |1-x| dx$

(g) $\int_0^1 (1-x^2)^n dx$

(c) $\int_{1/2}^2 \left(1+x-\frac{1}{x}\right) e^{x+\frac{1}{x}} dx$

(h) $\int_0^2 f(x) dx$

(d) $\int_0^{2\pi} x^2 \cos x dx$

$$\text{where } f(x) = \begin{cases} x^2 & \text{when } 0 \leq x \leq 1, \\ 2-x & \text{when } 1 < x \leq 2. \end{cases}$$

(e) $\int_0^{\ln 2} xe^{-x} dx$

8. Evaluate

(a) $\frac{d}{dx} \int_0^{x^2} \sqrt{1+t^2} dt$

(c) $\lim_{x \rightarrow 0} \frac{\int_0^x \cos t^2 dt}{x}$

(b) $\frac{d}{dx} \int_{\sin x}^{\cos x} \cos(\pi t^2) dt$

(d) $\lim_{x \rightarrow +\infty} \frac{\left(\int_0^x e^{t^2} dt\right)^2}{\int_0^x e^{2t^2} dt}$

End