Introduction

In this course, we plan to discuss the following (in that sequence): real nos., sequence, functions, derivatives, integration.

One main difference between this course and math1010 is the following. Here we will define mathematical terms more rigorously, using often times the so-called ($\epsilon - \delta$ language). We will also provide some background knowledge related to those concepts introduced.

The first thing we want to do is to introduce some important inequalities, because you will need them when defining limits in the future.

Intervals

- $[a, b], (a, b), (a, b], [a, b), (-\infty, b], (a, \infty), (-\infty, b), [a, \infty)$,
- $(-\infty, \infty)$ (also denoted by $\mathbb{R}$)

We start by making the following definitions or assumptions. (Here below, we are tacitly assuming the phrase: 'for all real nos. a, b'.)

$(A1)$: $a < b$ if and only if $b - a > 0$.

$(A1)'$: $a \leq b$ if and only if $b - a \geq 0$.

From the above definitions, one can easily show the following (we still number them by putting an 'A' before the number, although they are not assumptions.)

$(A2)$: $a < b \implies$ for any real no. c, $a + c < b + c$

$(A3)$: $a < b \implies$ for any real no. $c > 0$, $ac < bc$

$(A4)$: $a < b \implies$ for any real no. $c < 0$, $ac > bc$

Remark. You need to use 'pos. no. $\times$ pos. no. $=$ pos. no.', respectively, 'neg. no. $\times$ neg. no. $=$ pos. no.' or other similar properties when proving $(A3)$ and $(A4)$.

Other simple properties of real nos. are:

$(A5):\ a, b$ both positive or both negative, $a < b \implies 1/b < 1/a$

$(A6):\ a, b$ both nonnegative, $a \leq b \iff a^2 \leq b^2$.

Finally, we mentioned the following result:

Theorem. Let a, b be two rational numbers satisfying $a < b$, then we can construct infinitely many irrational numbers α between them, i.e.

There exist ∞-ly many irrat. nos. α satisfying $a < \alpha < b$.