THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1010 University Mathematics 2016-2017
Midterm Examination

Name (in print): ____

Student ID: _______________ Programme: ___________ Section: MATH1010____

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INSTRUCTIONS to students:

1. Answer all questions. Show work to justify all answers.
2. The examination lasts 90 minutes.
3. There are a total of 80 points.
4. Answer the questions in the space provided.

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FOR MARKERS' USE ONLY:

1	
2	
3	
4	
5	
6	
Total	/80 points

1. (12 marks) Evaluate the following.

(a) $\lim_{x \to 1} \dfrac{3 - x - x^2 - x^3}{1 - x} =$

(b) $\lim_{x \to +\infty} \dfrac{e^{2x} + x^3 \cos x}{e^{2x} - x^3 \sin x} =$

(c) $\lim_{x \to -\infty} (x + \sqrt{x^2 - 8x + 3}) =$

2. (16 marks) Find $\dfrac{dy}{dx}$ if

(a) $y = \dfrac{e^{2x}}{1+x}$

Solution:

(b) $y = \ln(2 + \sin(1 + 3x))$

Solution:

(c) $xy^2 + \cos(x + y) = 1$

Solution:

(d) $y = (\ln x)^x$

Solution:

3. (10 marks) Evaluate the following limits.

(a) $\lim_{x \to 0} \dfrac{\tan^{-1} x}{1 - \sqrt{1 - x}}$ ($\tan^{-1} x = \arctan x$ is the inverse of tangent.)

Solution:

(b) $\lim_{x \to 0} \left(\dfrac{1}{\ln(1 + x)} - \dfrac{1}{\sin x} \right)$

Solution:

4. (12 marks) Let a_n be the sequence defined by

$$\begin{cases} a_{n+1} = 3 - \dfrac{1}{a_n}, & \text{for } n \geq 1 \\ a_1 = 1. \end{cases}$$

(a) Show that $1 \leq a_n \leq 3$ for any $n \geq 1$.

(b) Show that $a_{n+1} - a_n > 0$ for any $n \geq 1$ by mathematical induction.

(c) Explain whether the limit of a_n exists and find the limit if it exists.

5. (15 marks) Let

$$f(x) = \begin{cases} x^2 \sin(\ln|x|), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

(a) Write down the first derivative of the function $\ln|x|$ for $x \neq 0$. No working steps or proof is required.

(b) Find $f'(x)$ for $x \neq 0$.

(c) Find $f'(0)$.

(d) Explain whether $f'(x)$ is differentiable at $x = 0$.

6 (15 marks) Let $f(x)$ be a function such that $f'(x)$ is strictly decreasing.

(a) Prove that $f'(x+1) < f(x+1) - f(x) < f'(x)$ for any x.

(b) Prove that

$$f'(1) + f'(2) + f'(3) < f(3) - f(0) < f'(0) + f'(1) + f'(2).$$

END OF PAPER