1. Compute

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{\arctan(x^2)}{1+x^2}\,dx.$$

   Hint: Substitute $u = \frac{1}{x}$, and show that if $I$ is the integral above, then

$$I = \frac{\pi}{2} \int_{1/\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\,dx - I.$$

2. Compute

$$\int \frac{1}{1+\sin x}\,dx.$$

   Hint: Multiply by $\dfrac{1-\sin x}{1-\sin x}$ (or use $t$-substitution, i.e. substitute $t = \tan\dfrac{x}{2}$)

3. Compute

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+(\tan x)^{\sqrt{5}}}\,dx.$$

   Hint: Substitute $u = \frac{\pi}{2} - x$, and use symmetry.

4. Compute

$$\int \cos(\ln x)\,dx.$$

   Hint: Integrate by parts twice, or use the substitution $u = \ln x$ to reduce to the more familiar integral $\int e^x \cos x\,dx$.

5. Compute

$$\int \frac{\sin x}{\sin x + \cos x}\,dx$$

   Hint: Either let

$$I = \int \frac{\sin x}{\sin x + \cos x}\,dx, \quad J = \int \frac{\cos x}{\sin x + \cos x}\,dx$$

   and compute both $I + J$ and $I - J$ (the latter can be computed using the substitution $u = \sin x + \cos x$), or write

$$\frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$$

   and substitute $u = \tan x$. Yet another way is $t$-substitution: substitute $t = \tan\frac{x}{2}$.

6. Compute

$$\int \frac{1}{\sin(x-a)\sin(x-b)}dx$$

if $a - b$ is not a multiple of $\pi$.

Hint:

$$\int \frac{1}{\sin(x-a)\sin(x-b)}dx = \frac{1}{\sin(a-b)}\int \frac{\sin[(x-b)-(x-a)]}{\sin(x-a)\sin(x-b)}dx.$$

7. Compute

$$\int \sqrt{1+e^{2x}}dx.$$

Hint: Substitute $u = \sqrt{1+e^{2x}}$. Or else, substitute $v = e^x$. Then one arrives at the integral

$$\int \frac{\sec^3 x}{\tan x}dx,$$

which one could integrate by using

$$\int \frac{\sec^3 x}{\tan x}dx = \int \frac{\sec^3 x \tan x}{\tan^2 x}dx = \int \frac{\sec^2 x}{\sec^2 x - 1}d(\sec x).$$

The integrand in the last integral is a rational function in $\sec x$, which can be integrated using partial fractions.

8. Compute

$$\int_0^1 \frac{\ln(1+x)}{1+x^2}dx$$

Hint: Let $I$ be the above integral. Substitute $x = \tan\theta$ to see that

$$I = \int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta.$$

Substitute $t = \frac{\pi}{4} - \theta$ to see that

$$I = \ln 2 \int_0^{\frac{\pi}{4}} dt - I$$

so $I = \frac{\pi}{8}\ln 2$.

9. Compute

$$\int_0^1 (1-x^7)^{1/5} - (1-x^5)^{1/7}dx.$$

Hint: The answer is 0; one just needs to show

$$\int_0^1 (1-x^7)^{1/5}dx = \int_0^1 (1-y^5)^{1/7}dy.$$

But both are the area bounded by the curve $x^7 + y^5 = 1$ with the $x$ and $y$-axes. So the two integrals are equal.