MATH4060 Exercise 3

Deadline: October 16, 2015.

The questions are from Stein and Shakarchi, Complex Analysis, unless otherwise stated.

Chapter 2. Exercise 11, 12.

Chapter 3. Exercise 15(a)(d), 19, 22.

Chapter 5. Exercise 8, 9, 10, 11, 13.

Additional Exercise. Show that if c > 0, then

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{a^s}{s(s+1)} ds = \begin{cases} 0 & \text{if } 0 < a \le 1\\ 1 - \frac{1}{a} & \text{if } a \ge 1. \end{cases}$$

Here the integral is over the vertical line Re s=c. (Hint: Consider the straight line contour from c-iR to c+iR. Complete this by a semi-circular arc centered at c and of radius R, either in the clockwise or the anti-clockwise direction, as appropriate.)