Note01-metric

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Let X be a nonempty set, a function
$d : X \times X \longrightarrow [0,\infty)$ is a metric if

(i) $d(x,y) = 0 \iff x = y$

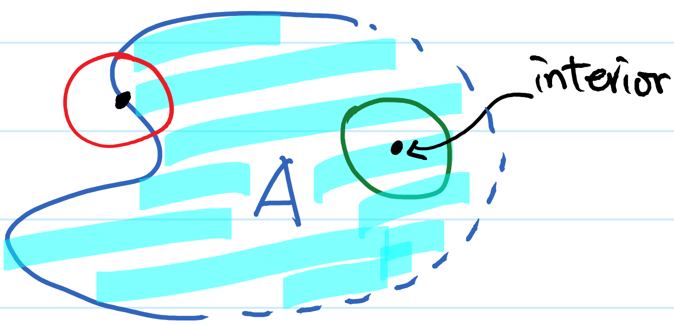
(ii) $d(x,y) = d(y,x)$

(iii) Δ-inequality
$$d(x,y) + d(y,z) \geq d(x,z)$$

In a metric space, a ball with
center $a \in X$ and radius $r > 0$ is
$$B(a,r) = \{x \in X : d(x,a) < r\}$$

The above are naturally understood when
$X = \mathbb{R}^n$, $d(x,y) = \|x-y\|$
$$= \left[\sum_{k=1}^{n} (x_k - y_k)^2\right]^{1/2}$$

Given any subset $A \subset X$, a point $x \in A$
is an interior point of A if
$\exists\ \delta > 0$ such that $x \in B(x,\delta) \subset A$



Discrete metric

$$d : X \times X \longrightarrow [0,\infty)$$

$$d(x,y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases}$$

Exercise.

What are $B(x,\frac{1}{2})$, $B(x,1)$, $B(x,2)$ where $x \in X$?

Exercise

For any $A \subset X$ and $x \in A$, x is an interior point of A.

In other words, every $A \subset X$ is an open set according to d.

i.e. $\{\text{open sets}\} = \mathcal{P}(X)$