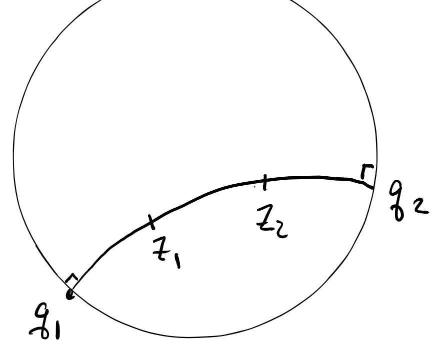
MMAT5120 HW 2 Due Date: Nov 23, 2020

(1) Show that the hyperbolic distance in the disk model satisfies $d(z_1, z_2) = \ln(z_1, z_2, q_2, q_1)$ where q_1, q_2 are determined as in the figure, and (z_1, z_2, q_2, q_1) is the cross ratio.



(2) Let C be the circumference of a hyperbolic circle with hyperbolic radius R. Show that

$$C = 2\pi \sinh(R).$$

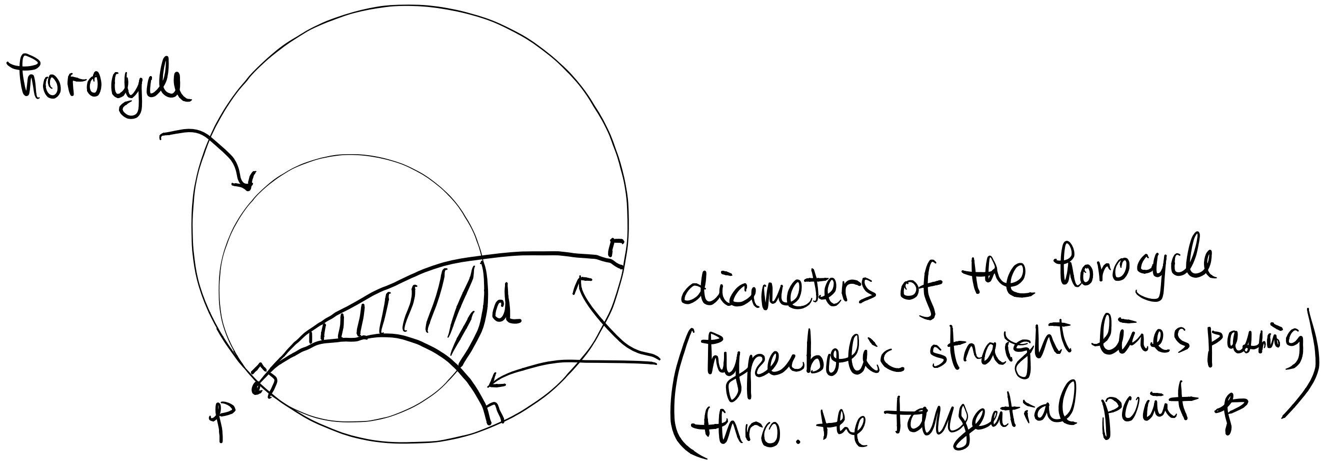
(3) Prove that any 2 horocycles in hyperbolic geometry are congruent.

(4) Let $S : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ be the Möbius transformation given by

$$w = Sz = i\,\frac{1+z}{1-z}.$$

Show that $T \in \mathbb{H} \iff S \circ T \circ S^{-1} \in \overline{\mathbb{H}}$, where $\mathbb{H}$ and $\overline{\mathbb{H}}$ are the groups of transformation of the disk model and upper half plane model of the hyperbolic geometry respectively.

(5) Suppose that the hyperbolic length of the arc on the horocycle between two diameter is d. Find the shaded area shown in the figure in terms of d.



(End)