MATH3060 HW7 Due date: Nov 20, 2020 (at 12:00 noon)

1. Let G be a bounded convex open subset of $\mathbb{R}^n$. Show that a family of equicontinuous functions is bounded in $C(\overline{G})$ if there exists a point $x_0 \in \overline{G}$ and a constant $M > 0$ such that $|f(x_0)| \le M$ for all f in the family.

2. Show that the sequence

$$f_n(x) = \int_0^x \frac{\sqrt{n}}{\sqrt{n+t^3}} e^{-nt^2} \, dt, \quad x \in [0,1]$$

has a convergent subsequence in $(C[0,1], d_\infty)$.

3. Show that for any fixed $M > 0$, every sequence in

$$\mathcal{E}_M = \left\{ f \in C^1[0,1] : f(0) = 0 \text{ and } \int_0^1 |f'(t)|^2 \, dt \le M \right\}$$

contains a convergent subsequence in $(C[0,1], d_\infty)$.

4. Suppose that $\sigma : [0, +\infty) \to \mathbb{R}$ is a continuous, nondecreasing function with $\sigma(0) = 0$. Show that

$$\mathcal{E}_\sigma = \{ f \in C[a,b] : |f(x) - f(y)| \le \sigma(|x-y|), \ \forall x, y \in [a,b] \}$$

is an equicontinuous family.

(End)