MATH3060 HW1 Due date: Sep 25, 2020 (at 12:00 noon)

1. Each of the following functions (on the left-hand side) are defined on $[-\pi, \pi]$. Sketch the 2π-periodic extension, find the corresponding Fourier expansion, and discuss the pointwise convergence (using Thm 1.5 and 1.6)

(a) $$f_1(x) = \begin{cases} x, & x \in [0, \pi] \\ 0, & x \in [-\pi, 0) \end{cases}$$

(b) $$f_2(x) = \begin{cases} -1, & x \in [0, \pi] \\ +1, & x \in [-\pi, 0) \end{cases}$$

(c) $f_3(x) = e^x$

(2) Show that the function $f(x) = |x|^\alpha$, $x \in [-\pi, \pi]$ is not Lipschitz continuous at $x = 0$ for any $0 < \alpha < 1$.

(3) Consider the function $f(x) = x$ on $(0, 2\pi]$ and its 2π-periodic extension $\tilde{f}(x)$ by

$$\tilde{f}(x) = f(x - 2k\pi) \quad \text{for } x \in (2k\pi, 2(k+1)\pi], \ \forall k \in \mathbb{Z}.$$

Sketch $\tilde{f}$, find its Fourier series, and discuss the pointwise convergence. Finally, if the Fourier series converges at the point $x=0$, what value does it limit to?

(Compare with $f(x) = x$ on $[-\pi, \pi]$)

(2) Consider the function $f(x) = \sin 2x$ on $(0, \pi]$ and extend to an even function $f_1(x)$ on $[-\pi, \pi]$, then further extend f_1 to a 2π-periodic function $\tilde{f}_1$ as usual. Sketch $\tilde{f}_1$. Show that

$$\tilde{f}_1 \sim \frac{8}{\pi} \sum_{k=0}^{\infty} \frac{1}{4-(2k+1)^2} \cos(2k+1)x.$$

Discuss the pointwise and uniform convergence.

(using Thm 1.5 and Thm 1.7. Compare with $\sin 2x$ on $[-\pi, \pi]$)

(End)