

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530A: Games & Strategic Thinking 2022-2023 Term 1
Homework Assignment 3
Due Date: December 7, 2021 (Wednesday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the “Submission Details” is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website <https://www.cuhk.edu.hk/policy/academichonesty/>

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature

Date

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, [here are a few tips](#).

Where is Gradescope?

Do the following:

1. Go to 2022R1 Games and Strategic Thinking (UGEB2530A)
 2. Choose Tools in the left-hand column
 3. Scroll down to the bottom of the page
 4. The green Gradescope icon will be there
- Late assignments will receive a grade of 0.
 - Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

For the declaration sheet:

Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence “I have read the university regulations.”

The deadline for HW3 is December 7, 2021 before 11:59 PM. Please plan ahead and submit your HW3 on time.

- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. Find the threat solutions of the games with the following bimatrices. That is, find the threat strategy and the payoff of each player.

(a) $\begin{pmatrix} (3, -2) & (2, 4) \\ (1, 0) & (3, -1) \end{pmatrix}$

(b) $\begin{pmatrix} (5, 3) & (1, 3) \\ (4, 5) & (2, 1) \end{pmatrix}$

2. The second term is starting soon. Joseph and Patrick plan to move to one of the four student hostels at New Asia College from their homes by a cargo van. If they each hire a cargo van, the costs for Joseph and Patrick are \$52 and \$94 respectively. If they hire a cargo van together, the cost will be \$104. By considering Shapley's values, find a suitable way for Joseph and Patrick to divide the cost if they hire a cargo van together.
3. Three ladies A , B and C have dinner together at a restaurant for their secondary school class reunion. Ladies A , B , and C have digital coupons which allow them to enjoy a 20%, 30% and 50% discount on the total bill respectively. Suppose the three discounts may be used at the same time and each of them planned to spend \$125 on the meal before discount. (A and B need to pay $\$250 \times (1-20\%) \times (1-30\%) = \140 if they go to dinner together.) By considering Shapley's values, find a suitable way for the three ladies to divide the total cost of the dinner. Please give your answers to 2 decimal places.
4. Three districts of Hong Kong: North District (N), Tao-Po District (P) and Sha Tin (T) District, are considering whether to rebuild a joint cycle track network. Interwoven across different recharge points along the way will be water machines and washrooms, but most importantly it will provide access to beautiful hikes and scenic views. The costs of the construction work are listed in the following table:

Coalition	Cost(in ten of millions of dollars)
$\{N\}$	12
$\{P\}$	6
$\{T\}$	7
$\{N, P\}$	15
$\{N, T\}$	14
$\{P, T\}$	11
$\{N, P, T\}$	20

- (a) Find $\nu(S)$ for each coalition S where ν is the characteristic function.
- (b) Find the Shapley's values for N , P , and T .
- (c) How should the three districts divide the construction costs?

Please give your answers to 2 decimal places.