THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530A: Games & Strategic Thinking 2022-2023 Term 1
Homework Assignment 2
Due Date: November 25, 2022 (Friday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature ____________________ Date ____________________

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. Go to 2022R1 Games and Strategic Thinking (UGEB2530A)
  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

  For the declaration sheet:

  Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

The deadline for HW2 is November 25, 2022 before 11:59 PM. Please plan ahead and submit your HW2 on time.

- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. Find all the pure Nash equilibria of the games with the following game bimatrices and state whether they are Pareto optimal.

   (a) $\begin{pmatrix} (1,3) & (4,6) \\ (2,5) & (1,3) \end{pmatrix}$

   (b) $\begin{pmatrix} (-1,2) & (3,6) & (1,-3) \\ (3,1) & (5,-1) & (4,2) \\ (6,3) & (-2,2) & (3,0) \end{pmatrix}$

2. Consider the 2-person game with the following bimatrix

$$\begin{pmatrix} (1,4) & (5,1) \\ (4,3) & (2,6) \end{pmatrix}$$

   (a) Find the prudential strategy of each player and the security levels of the players.

   (b) Find the Nash equilibrium of the game and the corresponding payoffs to the players.

3. Let us consider the 2-person game used by a non-profit organisation (social hut) that wishes to aid a pauper if he searches for work but not otherwise, and a pauper who searches for work only if he cannot depend on organisation aid, and who may not succeed in finding a job even if he tries. The payoffs are $5, -2$ (for organisation, pauper) if the organisation aids and the pauper tries to work; $1, 3$ if the organisation does not aid and the pauper tries to work; $4, 2$ if the organisation aids and the pauper does not try to work; and $-3, 0$ in the remaining case.

   (a) Write down the game bimatrix.

   (b) Find the prudential strategy of each player and the security levels of the players.

   (c) Find the Nash equilibrium of the game and the corresponding payoffs to the players.

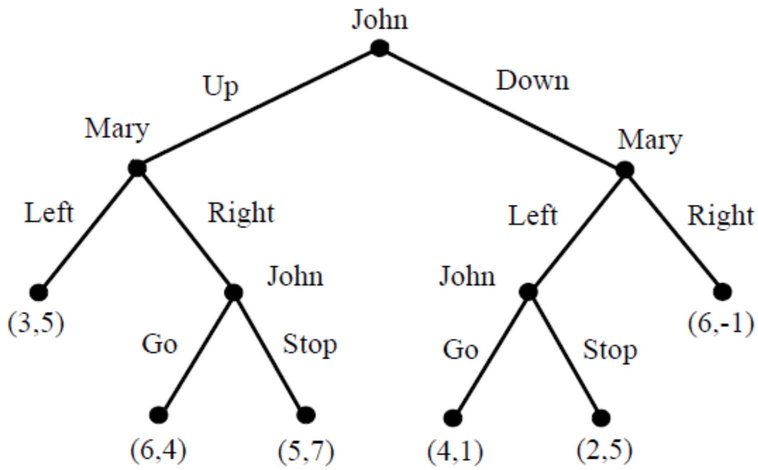
4. Two technology companies A and B plan to produce two 5G (Fifth-generation wireless) products. Each of the companies may produce either a 5G modem or a 5G New Radio millimeter. If both companies produce a 5G modem, each of the companies would have a profit of \$5 million. If company A produces a 5G modem and company B produces a 5G New Radio millimeter, the profits of A and B will be \$9 million and \$12 million respectively. If A produces a 5G New Radio millimeter and B produces a 5G modem , the profits of A and B will be \$13 million and \$11 million respectively. If both companies produce a 5G New Radio millimeter, the profits of A and B will be \$7 million and \$8 million, respectively.

   (a) Write down the game bimatrix. (Payoffs to A and B in millions of dollars)

   (b) Find the prudential strategies and the security levels of the two companies.

   (c) Find all the Nash equilibria and the corresponding payoffs to the two companies.

5. Solve the following game tree by backward induction and write down the payoff pair in the solution.

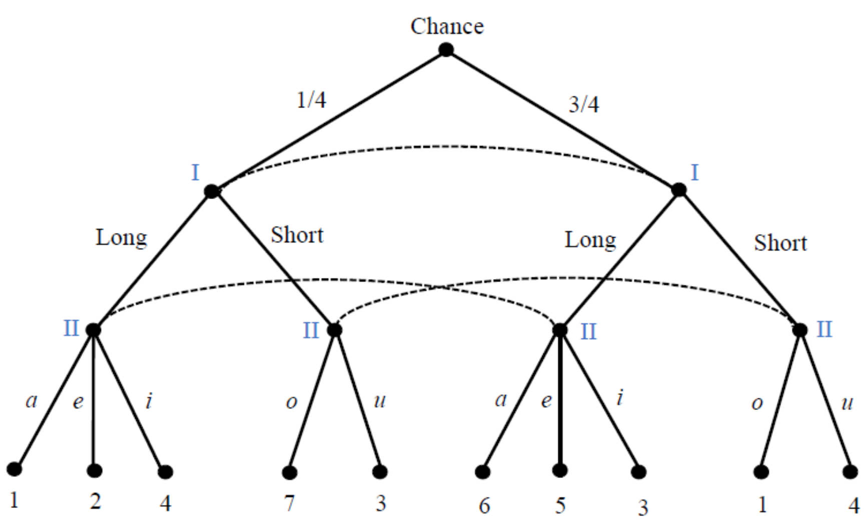


6. Eve and Noa start with \$10 in each of their piles. They take turns choosing one of two actions, continue or stop, with Eve choosing first. Each time a player says continue, \$10 will be removed from her pile, and \$20 will be added to the other player's pile. The game automatically stops when the total amount in their piles reaches \$60.

   (a) Draw the game tree of the game.

   (b) Solve the game by backward induction and write down the payoffs of the players in the solutions.

7. Consider a zero sum game between Player I and Player II with game tree



The numbers assigned to the terminal nodes are the payoffs to Player I.

   (a) Write down all strategies of Player I and Player II.

   (b) Find the strategic form (game matrix) of the zero sum game.

8. Oscar chooses a number from 1, 4 and 7. Lucy, who knows whether the chosen number is odd or even but does not know the exact value, must choose between 2 or 5. Then Lucy pays Oscar with an amount equal to the (absolute) difference of the numbers.

   (a) Draw the game tree of the game.

   (b) Write down all strategies of Oscar and Lucy.

   (c) Find the maximin strategy of Oscar, the minimax strategy of Lucy and the value of the game.

9. Players I and II play the following bluffing game. Each player bets \$1. Player I is given a card which is high or low; each is equally likely. Player I sees the card, player II doesn't. Player I can raise the bet to \$2 or fold. If player I folds, player I loses \$1 to player II. If player I raises, player II can call or fold. If player II folds, he loses \$1 to player I no matter what the card is. If player II calls, then player I wins \$2 from player II if his card is high and loses \$2 to player II if the card is low.

   (a) Draw the game tree of the game.

   (b) Write down all strategies of players I and II.

   (c) Write down the strategic form (game matrix) of the game.

   (d) Solve the game.

10. Anna has two coins. One is fair (probability 1/2 of heads and 1/2 of tails) and the other is biased with probability 1/4 of heads and 3/4 of tails. Anna knows which coin is fair and which is biased. She selects one of the coins and tosses it. The outcome of the toss is announced to Elsa. Then Elsa must guess whether Anna chose the fair or biased coin. If Elsa is incorrect, she pays \$2 to Anna, and if she is correct, she receives \$2 from Anna.

    (a) Draw the game tree.

    (b) Write down all strategies of Anna and Elsa.

    (c) Write down the strategic form of the game.

    (d) Solve the game.