**THE CHINESE UNIVERSITY OF HONG KONG**
**Department of Mathematics**
**UGEB2530A: Games & Strategic Thinking 2022-2023 Term 1**
**Homework Assignment 1**
**Due Date: 17 October, 2022 (Monday) before 11:59 PM**

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

____________________ ____________________
Signature Date

## General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. Go to 2022R1 Games and Strategic Thinking (UGEB2530A)
  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

  For the declaration sheet:

  Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction).

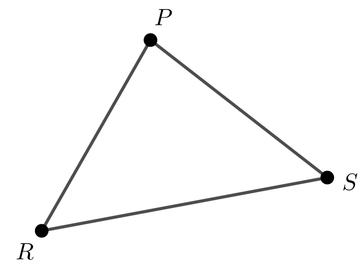
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. Evaluate the following matrix products.

(a) $\begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 6 & 5 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \begin{pmatrix} 1 & 3 \end{pmatrix}$

2. Let $A$ be the incident matrix of the graph




(a) Write down the matrix $A$.

(b) Find $A^2$ and $A^3$.

(c) Suppose David, Hank and Justin pass a ball among them and in the beginning the ball is on David's hand. It is given that

$$A^n = \frac{1}{3} \begin{pmatrix} 2^n + 2(-1)^n & 2^n - (-1)^n & 2^n - (-1)^n \\ 2^n - (-1)^n & 2^n + 2(-1)^n & 2^n - (-1)^n \\ 2^n - (-1)^n & 2^n - (-1)^n & 2^n + 2(-1)^n \end{pmatrix}.$$

(Those who know mathematical induction may try to prove this formula but it is not required.)

(i) Find the number of ways for David to pass the ball to Hank in 15 passes.

(ii) Find the number of ways for David to pass the ball back to himself in 15 passes.

3. Consider the bimatrix of the game:

$$(A, B) = \begin{pmatrix} (4, -1) & (2, 3) \\ (0, 5) & (3, 1) \end{pmatrix}.$$

(a) Suppose Player I uses $(0.2, 0.8)$ and Player II uses $(0.6, 0.4)$. Find the expected payoffs of the two players.

(b) Suppose Player I uses $(0.2, 0.8)$. Find the best strategy for Player II.

(c) Suppose Player II uses $(0.6, 0.4)$. Find the best strategy for Player I.

4. There are two boxes labeled A and B. Box A has a \$1 coin in it and Box B has a \$2 coin in it. Mary chooses one of the boxes and secretly triples the amount in the box. Peter chooses one box, without knowing what Mary has chosen, and gets the money inside. Then Mary gets the money in the other box.

(a) Write down the bimatrix of the game. (Use Mary as the row player and Peter as the column player.)
(b) Suppose Mary chooses Box A with a probability of 0.2 and Peter chooses Box A with a probability of 0.3. Find the expected payoffs of the two players.
(c) Suppose Mary chooses Box A with a probability of 0.2. Find the best strategy for Peter.
(d) Suppose Peter chooses Box A with a probability of 0.3. Find the best strategy for Mary.

5. In a rock-paper-scissors game, the loser pays the total number of fingers in the two gestures to the winner. The payoff is 0 if there is a draw.

(a) Write down the game matrix (payoff of player 1) of the game. (Use rock, paper, scissors, as the order of strategies.)
(b) Suppose player 1 uses $(0.2, 0.4, 0.4)$ and player 2 uses $(0.3, 0.5, 0.2)$. Find the expected payoff of player 1.
(c) If player 1 uses $(0.2, 0.4, 0.4)$, what is the best strategy for player 2.
(d) If player 2 uses $(0.3, 0.5, 0.2)$, what is the best strategy for player 1.

6. In a game, two players call out one of the numbers 1, 2, or 3 simultaneously. Let $S$ be the sum of the two numbers. If $S$ is even, then player 2 pays $S$ dollars to player 1. If $S$ is odd, then player 1 pays $S$ dollars to player 2.

(a) Write down the game matrix for the payoff of player 1.
(b) Write down the game matrix for the payoff of player 2.
(c) Find the expected payoff of player 1 if player 1 calls out the numbers 1, 2, 3 with probabilities 0.5, 0.3, 0.2 respectively, and player 2 calls out the numbers 1, 2, 3 with probabilities 0.1, 0.4, 0.5 respectively.
(d) Suppose player 2 calls out the numbers 1,2,3 with probabilities 0.1, 0.4, 0.5 respectively. What is the best strategy for player 1 and what is his expected payoff if he uses this strategy?

7. For each of the following game matrices, determine whether there is a saddle point. Copy the game matrix and circle all saddle points of the matrix if there are any.

(a) $\begin{pmatrix} -1 & -4 & 5 & -2 \\ -3 & 5 & -1 & 0 \\ 2 & 4 & -1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & 7 & -3 & 0 \\ 0 & -4 & -1 & -3 \\ 2 & 4 & 6 & 4 \\ -2 & 2 & 3 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 2 & 5 & 2 \\ 2 & 1 & -1 & -20 \\ 3 & 2 & 4 & 2 \\ -20 & 0 & 18 & 1 \end{pmatrix}$

8. A company is trying to manufacture a new type of toy. The company has to decide whether to bring out a full, semi, partial or minimal product line. There are five levels of product acceptance and the company has estimated their probability of occurrence. Management of the company will make the decision on the basis of maximizing the expected profit from the first year of production. The relevant data is shown in the following table, where the first year profits are given in thousands of Hong Kong Dollars (HKD):

| | | Course of action | | | | |
|---|---|---|---|---|---|---|
| | | Excellent | Good | Fair | Poor | Very Poor |
| Product | Full | 2 | 4 | 3 | 8 | 4 |
| | Semi | 5 | 6 | 3 | 7 | 8 |
| | Partial | 6 | 7 | 9 | 8 | 7 |
| | Minimal | 4 | 2 | 8 | 4 | 3 |

Use the dominated strategy to obtain the optimal strategy (strategies) for the product line and course of action with its probability and determine the value of the game if any.

9. Solve the zero sum games, that is, find a maximin strategy for the row player, a minimax strategy for the column player and the value of the game, with the following game matrices.

(a) $\begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix}$

10. Solve the zero sum games with the following game matrices.

(a) $\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 6 \\ 0 & 4 \\ 2 & 3 \\ 3 & 1 \end{pmatrix}$

11. Solve the zero sum game with game matrix

$$\begin{pmatrix} 5 & 3 & 8 & 1 \\ 2 & 3 & 5 & 10 \\ 7 & 5 & 6 & 2 \\ 6 & 4 & 3 & 1 \end{pmatrix}$$

12. Solve the following game matrices.

(a) $$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

(b) $$\begin{pmatrix} 2 & -4 & -4 \\ -4 & 6 & -4 \\ -4 & -4 & 16 \end{pmatrix}$$