i, (you can consider $\hat{p}$ be a $\vec{p}$ or a <u>unit</u> vector. But it need to be consistent throughout the whole question)

let the vector be $v = (v_1, v_2, v_3)$

$v \parallel \vec{p} \times \vec{q}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 1 & -3 & -1 \end{vmatrix} = (6, 3, -3)$$

$$\therefore v = (6, 3, -3) / (6^2 + 3^2 + (-3)^2)^{\frac{1}{2}} = \left(\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6}, \frac{-\sqrt{6}}{6} \right)$$

ii, consider $\hat{p}$ be the same as $\vec{p}$

$$\vec{p} \times \hat{q} = (6, 3, -3)$$

$$(\vec{p} \times \vec{q}) \cdot i = 6$$

$$\vec{p} \times j = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 1 & 0 \end{vmatrix} = (-1, 0, 2)$$

$$(\vec{p} \times j) \cdot \vec{q} = (-1, 0, 2) \cdot (1, -3, -1)$$

$$= -1 \times 1 + 0 \times (-3) + 2 \times (-1)$$

$$= -3$$

$$\therefore (\vec{p} \times \vec{q}) \cdot i - (\vec{p} \times j) \cdot \vec{q}$$

$$= 6 - (-3) = 9$$

consider $\hat{p}$ be unit vector.

$$\hat{p} = (2, -3, 1) / (2^2 + (-3)^2 + 1^2)^{1/2} = \left(\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$$

$$\hat{q} = (1, -3, -1) / (1^2 + (-3)^2 + (-1)^2)^{1/2} = \left(\frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\right)$$

$$\hat{p} \times \hat{q} = \begin{vmatrix} i & j & k \\ \frac{2}{\sqrt{14}} & \frac{-3}{\sqrt{14}} & \frac{1}{\sqrt{14}} \\ \frac{1}{\sqrt{11}} & \frac{-3}{\sqrt{11}} & \frac{-1}{\sqrt{11}} \end{vmatrix} = \left(\frac{6}{\sqrt{154}}, \frac{3}{\sqrt{154}}, \frac{-3}{\sqrt{154}}\right)$$

$$(\hat{p} \times \hat{q}) \cdot i = \frac{6}{\sqrt{154}}$$

$$\hat{p} \times j = \begin{vmatrix} i & j & k \\ \frac{2}{\sqrt{14}} & \frac{-3}{\sqrt{14}} & \frac{1}{\sqrt{14}} \\ 0 & 1 & 0 \end{vmatrix} = \left(\frac{-1}{\sqrt{14}}, 0, \frac{2}{\sqrt{14}}\right)$$

$$(\hat{p} \times j) \cdot \hat{q} = \frac{-1}{\sqrt{14}} \times \frac{1}{\sqrt{11}} + 0 \times \frac{-3}{\sqrt{11}} + \frac{2}{\sqrt{14}} \times \frac{-1}{\sqrt{11}}$$

$$= \frac{-3}{\sqrt{154}}$$

$$\therefore \ (\hat{p} \times \hat{q}) \cdot i - (\hat{p} \times j) \cdot \hat{q}$$

$$= \frac{6}{\sqrt{154}} - \frac{-3}{\sqrt{154}} = \frac{9}{\sqrt{154}}$$

2. It is true.

Let $u = (u_1, u_2, u_3)$. $V = (V_1, V_2, V_3)$

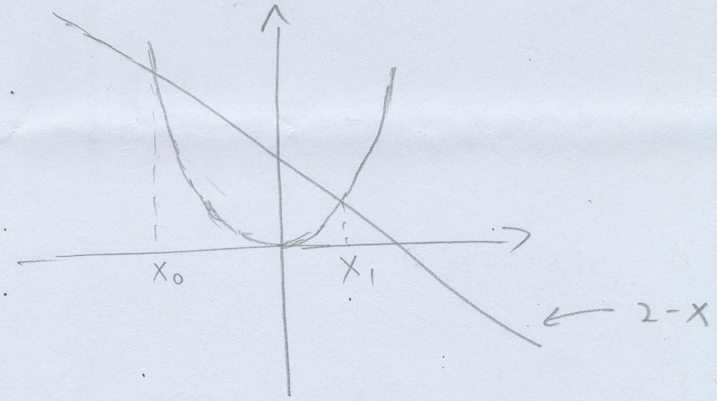
$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= (u_2 v_3 - u_3 v_2 ,\ u_3 v_1 - u_1 v_3 ,\ u_1 v_2 - u_2 v_1)$$

$$u \cdot (u \times v) = u_1(u_2 v_3 - u_3 v_2) + u_2(u_3 v_1 - u_1 v_3) + u_3(u_1 v_2 - u_2 v_1)$$

$$= 0$$

3.



$$\begin{cases} y = x^2 \\ y = 2-x \end{cases}$$

$\Rightarrow x^2 = 2-x \Rightarrow (x+2)(x-1) = 0$

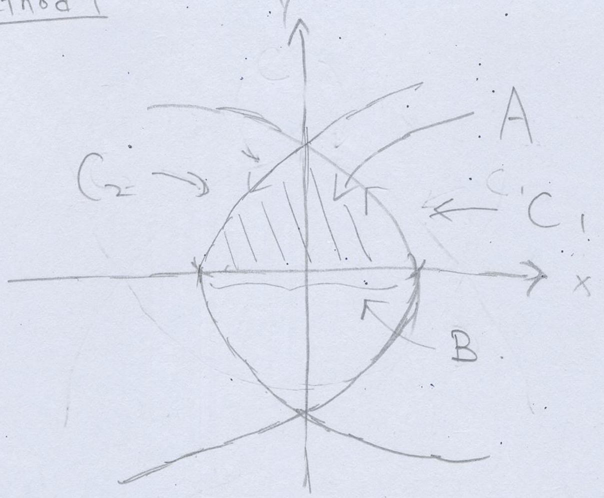
$\Rightarrow x_0 = -2 , x_1 = 1$

the area $= \int_{x_0}^{x_1} (2-x) - (x^2)\, dx$

$$= \int_{-2}^{1} 2 - x - x^2 \, dx$$

$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^{1} = 9/2$$

4. Method 1



let $F = (u, v)$

$$\int_B F \cdot dv + \int_C F \cdot dv = \int_A \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \, dx \, dy$$

B: $(x, y) = (x, 0)$ x from -1 to 1

$$\int_B F \cdot dv = \int_{-1}^{1} \left(x \times 0 \times \cos x , \; x^2 - 0 \times e^0 \right) \cdot (1, 0) \, dx$$

$$= \int_{-1}^{1} 0 \, dx = 0$$

$$\int_A \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \, dx \, dy = \int_A 2x - x\cos x \, dx \, dy$$

$$= \int_0^1 \int_{y^2-1}^{1-y^2} 2x - x\cos x \, dx \, dy$$

$$= \int_0^1 \left(\left[x^2 \right]_{y^2-1}^{1-y^2} - \int_{y^2-1}^{1-y^2} x \, d\sin x \right) dy$$

$$= \int_0^1 \left(0 - \left[x \sin x \right]_{y^2-1}^{1-y^2} + \int_{y^2-1}^{1-y^2} \sin x \, dx \right) dy$$

$$= \int_0^1 0 - 0 + \left[-\cos x \right]_{y^2-1}^{1-y^2} dy$$

$$= 0 \implies \int_C F \cdot dv = 0$$

Method 2

$C_1 : (x, y) = (1-t, \sqrt{1-(1-t)})$ t from 0 to 1
$= (1-t, \sqrt{t})$

reverse the curve C_2 to $\overline{C_2}$

$\overline{C_2} : (x, y) = (t-1, \sqrt{1+(t-1)})$ t from 0 to 1
$= (t-1, \sqrt{t})$

$$\int_{C_2} F \cdot dv = - \int_{\overline{C_2}} F \cdot dv$$

$$\therefore \int_C F \cdot dv = \int_{C_1} F \cdot dv + \int_{C_2} F \cdot dv$$

$$= \int_{C_1} F \cdot dv - \int_{\overline{C_2}} F \cdot dv$$

$$= \int_0^1 \left((1-t)\sqrt{t}\cos(1-t), (1-t)^2 - \sqrt{t}e^{\sqrt{t}}\right) \cdot \left(-1, \frac{1}{2\sqrt{t}}\right) dt$$

$$- \int_0^1 \left((t-1)\sqrt{t}\cos(t-1), (t-1)^2 - \sqrt{t}e^{\sqrt{t}}\right) \cdot \left(1, \frac{1}{2\sqrt{t}}\right) dt$$

as $\begin{cases} (1-t)\sqrt{t}\cos(1-t) = -(t-1)\sqrt{t}\cos(t-1) \\ (1-t)^2 - \sqrt{t}e^{\sqrt{t}} = (t-1)^2 - \sqrt{t}e^{\sqrt{t}} \end{cases}$

$$\therefore \int_C F \cdot dv = 0$$

5i, by Green's Thm

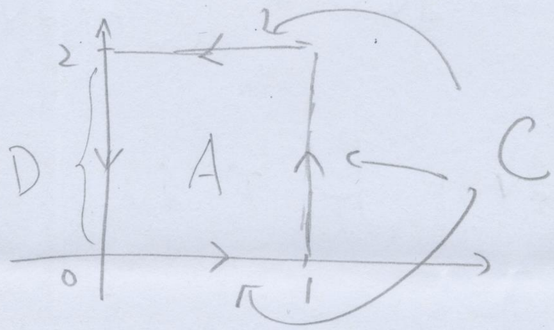
$$\int_C F \cdot dr = \int_{\text{the ellipse}} \frac{\partial x}{\partial x} - \frac{\partial (0)}{\partial y} \, dx\,dy$$

$$= \int_{\text{the ellipse}} 1 \, dx\,dy = \text{area of the ellipse}.$$

$\therefore \int_C F \cdot dr > 0$.

$\therefore$ it is true.

5ii,



By green's thm,

$$\int_C F \cdot dr + \int_D F \cdot dr = \int_A Q_x - P_y \, dy\,dx$$

$$\int_C F \cdot dr = \int_0^1 \int_0^2 Q_x - P_y \, dy\,dx - \int_2^0 (P, Q) \cdot (0, \overset{\substack{\therefore \text{ parametrized by} \\ (0,y)}}{\downarrow} 1) \, dy$$

$$= \int_0^1 \int_0^2 Q_x - P_y \, dy\,dx + \int_0^2 Q(0, y) \, dy.$$

it is true