THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5520 Differential Equation & Linear Algebra

Assignment 5
Due date: No need to submit

Exercise 6.2

1. Find the general solutions to the following systems of differential equations.

(a) $\begin{cases} x_1' = x_1 + 2x_2 \\ x_2' = 2x_1 + x_2 \end{cases}$ (c) $\begin{cases} x_1' = x_1 - 5x_2 \\ x_2' = x_1 - x_2 \end{cases}$ (f) $\begin{cases} x_1' = 4x_1 + x_2 + x_3 \\ x_2' = x_1 + 4x_2 + x_3 \\ x_3' = x_1 + x_2 + 4x_3 \end{cases}$

2. Solve the following initial value problem.

(b) $\begin{cases} x_1' = 9x_1 + 5x_2 \\ x_2' = -6x_1 - 2x_2 \\ x_1(0) = 1,\ x_2(0) = 0 \end{cases}$

3. Solve $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the given matrix $\mathbf{A}$.

(b) $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 5 & -1 \end{pmatrix}$ (d) $\mathbf{A} = \begin{pmatrix} 4 & -1 & -1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

Exercise 6.3

1. Find the general solution to the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the given matrix $\mathbf{A}$.

(a) $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$ (d) $\mathbf{A} = \begin{pmatrix} -3 & 0 & -4 \\ -1 & -1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$

(c) $\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ (f) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & -2 & -3 \\ 2 & 3 & 4 \end{pmatrix}$

Exercise 6.4

1. Find $\exp(\mathbf{A}t)$ where $\mathbf{A}$ is the following matrix.

(b) $\begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}$ (f) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 0 & 3 \\ 0 & 0 \end{pmatrix}$ (g) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

2. Solve the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ with initial condition $\mathbf{x}(0) = \mathbf{x}_0$ for given $\mathbf{A}$ and $\mathbf{x}_0$.

   (a) $\mathbf{A} = \begin{pmatrix} 2 & 5 \\ -1 & -4 \end{pmatrix}$; $\mathbf{x}_0 = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

   (c) $\mathbf{A} = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$; $\mathbf{x}_0 = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$

Exercise 6.5

1. For the given matrix $\mathbf{A}$, find the Jordan normal form of $\mathbf{A}$ and the matrix exponential $\exp(\mathbf{A}t)$.

   (a) $\begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$

   (c) $\begin{pmatrix} 5 & -1 & 1 \\ 1 & 3 & 0 \\ -3 & 2 & 1 \end{pmatrix}$

   (d) $\begin{pmatrix} -2 & -9 & 0 \\ 1 & 4 & 0 \\ 1 & 3 & 1 \end{pmatrix}$

   (e) $\begin{pmatrix} -1 & 1 & 1 \\ 1 & 2 & 7 \\ -1 & -3 & -7 \end{pmatrix}$

Exercise 6.6

1. Find a fundamental matrix for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ where $\mathbf{A}$ is the following matrix.

   (a) $\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$

   (c) $\begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}$

   (f) $\begin{pmatrix} 1 & -3 \\ 3 & -5 \end{pmatrix}$

   (g) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{pmatrix}$

   (j) $\begin{pmatrix} 3 & 1 & 3 \\ 2 & 2 & 2 \\ -1 & 0 & 1 \end{pmatrix}$

2. Find the fundamental matrix $\mathbf{\Phi}$ which satisfies $\mathbf{\Phi}(0) = \mathbf{\Phi}_0$ for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for the given matrices $\mathbf{A}$ and $\mathbf{\Phi}_0$.

   (a) $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}$; $\mathbf{\Phi}_0 = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$

   (c) $\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ -4 & 7 & -4 \\ -2 & 2 & 1 \end{pmatrix}$; $\mathbf{\Phi}_0 = \begin{pmatrix} 2 & 0 & -1 \\ 0 & -3 & 1 \\ -1 & 1 & 0 \end{pmatrix}$

Exercise 6.7

1. Use the method of variation of parameters to find a particular solution for each of the following non-homogeneous equations.

   (a) $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\mathbf{x} + \begin{pmatrix} -6e^{5t} \\ 6e^{5t} \end{pmatrix}$

   (c) $\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ 9e^{t} \end{pmatrix}$