THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3070 (Second Term, 2016–2017)
Introduction to Topology
Exercise 0 Preparation (Set Language)

Remarks

These exercises may give you an impression of the foundation needed in this course.

1. Let $f\colon X \to Y$ and $g\colon Y \to Z$; $A \subset X$, $B \subset Y$; if needed, $f(A) \subset B$. Determine the correctness of the following statements. Justify with proofs or counter-examples.

   (a) $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$

   (b) if $B_1 \subset B_2$ then $f^{-1}(B_1) \subset f^{-1}(B_2)$

   (c) if $A_1 \subset A_2$ then $f(A_2 * A_1) = f(A_2) * f(A_1)$ where $*$ may be $\cup$, $\cap$, $\setminus$ (set minus), or $\triangle$ (symmetric difference).

2. Define a relation $\sim$ on $\mathbb{R}^2$ by $(x_1, y_1) \sim (x_2, y_2)$ if $x_1^2 - y_1^2 = x_2^2 - y_2^2$. Show that this is an equivalence relation. What are its equivalence classes?

   For an equivalence relation $\sim$ (not necessarily the above) on a set X, what is its quotient map q defined on X?

   Under what condition does a function $f: W \to X/\sim$ has another $\tilde{f}: W \to X$ such that $f = q \circ \tilde{f}$?

3. Define a family of sets X_α for $\alpha \in A$ (index set) and the arbitrary product $\prod_{\alpha\in A} X_\alpha$.

   If there are functions $f_\alpha\colon X_\alpha \to Y$, is it possible to define a function $f\colon \prod_{\alpha\in A} X_\alpha \to Y$?

   On the other hands, if there are functions $g_\alpha\colon U \to X_\alpha$, is it possible to define a function $g\colon U \to \prod_{\alpha\in A} X_\alpha$?

4. Let $A_\alpha \subset X$ where $\alpha \in A$. Define $\bigcup_{\alpha\in A} A_\alpha$ and $\bigcap_{\alpha\in A} A_\alpha$.

   For $B \subset A$, what is the meaning of $\bigcup \{ A_\alpha : \alpha \in B \}$? What is the meaning of all arbitrary unions of sets in $\{ A_\alpha : \alpha \in A \}$?

   Let $\mathcal{C}$ be a set of sets. What is the notation $\bigcup \mathcal{C}$? What is $\bigcup \mathcal{B}$ where $\mathcal{B} \subset \mathcal{C}$?

5. What is a countable or uncountable set? State some propositions about countability between a set and its image under a function.

6. What are the basic requirements of an algebraic group?

   Give two examples of infinite group except $\mathbb{Z}$ and $\mathbb{R}$. Also, give two examples of finite non-abelian group.