MATH 3060 HW 9 No need to hand in

1. Let Ω be a bounded convex subset of $\mathbb{R}^n$. Show that a family of equicontinuous functions is bounded in $C(\Omega)$ if there exists a point $x_0 \in \Omega$ and a constant $M > 0$ such that $|f(x_0)| \leq M$ for all f in the family.

2. Let $\{f_n\}$ be a sequence in $C(\Omega)$ where Ω is open in $\mathbb{R}^n$. Suppose that on every compact subset of Ω, it is equicontinuous and bounded. Show that there is a subsequence $\{f_{n_j}\}$ converging to some $f \in C(\Omega)$ uniformly on every compact subset of Ω.

(Hint: Show that $\Omega = \bigcup_{i=1}^{\infty} K_i$, where K_j are compact subsets of Ω and $K_i \subset K_{i+1}$, $\forall i$.)

3. Let $K \in C([a,b] \times [a,b])$, and for $f \in C[a,b]$, define Tf by

$$(Tf)(x) = \int_a^b K(x,y) f(y)\, dy.$$

(a) Show that $Tf \in C[a,b]$.

(b) Show that if $\{f_n\}$ is a bounded sequence in $C[a,b]$, then $\{Tf_n\}$ contains a convergent subsequence.

4. Show that the boundary of a nonempty open set in a metric space must be closed and nowhere dense. Conversely, every closed nowhere dense set is the boundary of some open set.

5. Use Baire Category Theorem to show that transcendental numbers are dense in the set of real numbers.