MATH 3060 HW 8 Due Date: Nov 28, 2016

1. Let (X, d) be a metric space and $(Y, \|\cdot\|_Y)$ a Banach space. Show that the vector space $C_b(X;Y)$ consisting of all bounded continuous mapping from X to Y forms a Banach space under the supnorm
$$\|f\|_\infty = \sup_{x \in X} \|f(x)\|_Y .$$

2. Let D be a dense set in a complete metric space X. Show that every uniform continuous function defined on D can be extended to become a uniform continuous function on X.

3. Let l^∞ consist of all bounded sequences in $\mathbb{R}$. It is a Banach space under the norm $\|a\|_\infty = \sup_k |a_k|$ for $a = \{a_k\}$ in l^∞. Show that this space is not separable.

4. Show that the closure of totally bounded set is totally bounded.

5. Show that every totally bounded set is bounded but that the converse is not ture.