THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050B Mathematical Analysis I

Tutorial 4 (October 3)

The following were discussed in the tutorial this week:

1. Subsequences, Bolzano Weierstrass Theorem

2. Show that if (a_n) is a bounded divergent sequence, then (a_n) have at least two subsequences converging to different limits.

3. Let (x_n) be a sequence of real numbers. Define

$$\sigma_n = \frac{x_1 + x_2 \cdots + x_n}{n} \quad \text{for all } n \in \mathbb{N}.$$

   (a) If $\lim_n x_n = \ell$, where $\ell \in \mathbb{R}$, show that $\lim_n \sigma_n = \ell$.

   (b) Is the converse of (b) true?

4. Let (x_n) be a sequence of positive real numbers. Suppose $\lim_n \sqrt[n]{x_n} = L$, where L is a non-negative real number.

   (a) If $0 \leq L < 1$, show that $\lim_n x_n = 0$.

   (b) If $L > 1$, show that (x_n) is divergent.

   (c) What if $L = 1$?

5. Let (x_n) be a sequence of positive real numbers.

   (a) Suppose $\lim_n \frac{x_{n+1}}{x_n} = L$, where L is a non-negative real number. Show that $\lim_n \sqrt[n]{x_n} = L$.

   (**Hint:** You may assume the properties of exponential and logarithmic functions and use the result of 3(a).)

   (b) Is the converse of (a) true?