THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMAT5220 Complex Analysis and its Applications 2014-2015
Test 1, 11 Feb, 2015

- Time allowed: 45 minutes
- Answer all questions.
- Show your work clearly and concisely in your answer book.
- Write down your name and student ID number on the front page of your answer book.
- You are allowed to use a calculator in this test.

1. (a) If $f'(z) = 0$ for every $z \in \mathbb{C}$, by considering the Cauchy-Riemann equations, show that $f(z)$ is a constant.

   (b) By considering the derivative of the function $f(z) = \sin^2 z + \cos^2 z$, prove that

$$\sin^2 z + \cos^2 z \equiv 1.$$

(25 points)

2. Solve

   (a) $\sin z + \cos z = 2$.

   (b) $\operatorname{Log}(e^z) = 2$.

(25 points)

3. Suppose that C is the circle $|z| = 2$ oriented in the counterclockwise direction. By using ML-estimate, show that

$$\left| \int_C \frac{e^z}{z^2+1}\, dz \right| \leq \frac{4\pi e^2}{3}.$$

(25 points)

4. Let $f(z) = f(x + yi) = \sqrt{|xy|}$.

   (a) Show that the Cauchy-Riemann equations are satisfied at the point $(0, 0)$.

   (b) Show that f is not differentiable at $z = 0$.

(25 points)