

MATH 2550
Mid-Term Test (Prep.)

(Keywords: vectors in \mathbb{R}^2 or \mathbb{R}^3 , equations of planes, lines, tangent line to curves, to surfaces, equation of tangent line to a curve, equation(s) of a surface, normal vector, partial derivative, definite integral & area “under $y = f(x)$ ”, “area under $y = f(x)$ as an infinite sum, Riemann Sum, $f(\xi_i)$, Δx_i , line integral of a scalar field, line integral of a vector field, Green’s Theorem for a rectangle, Green’s Theorem in general, orientation of a curve.)

In this set of exercise we hope you can revise stuff learned so far. (I will put them on WeBWork for you to practice).

1. Consider the plane which passes through the point with coordinates $(0,1,1)$ and has normal vector $(1, -1, -1)^t$. Write down the equation of this plane in the form $(\vec{r} - \vec{r}_0) \cdot \hat{N} = 0$. (Here the notation \hat{N} means the “unit length” normal vector in the direction of the vector \vec{N}).
2. Rewrite the equation in question 1 in the form $x + By + Cz + D = 0$.
3. Find the equation of the plane containing the one point with the position vector given by $(1,1,1)^t$ and parallel (try to imagine what this means!!!) to the two (displacement) vectors $(0,1,0)^t$ and $(0,0,1)^t$.
4. Compute the distance between the two position vectors $(1,2,3)^t$ and $(-1,0,2)^t$.
5. Consider the square-based pyramid formed by joining the five points with position vectors $(0,0,1)^t, (1,1,0)^t, (-1, -1,0)^t, (1, -1,0)^t$ and $(1, -1,0)^t$. For each of the “slanted” faces, find an “outward” pointing “unit” normal vector.
6. Compute $\int_C [2xy \hat{i} + (x^2 - y^2)\hat{j}] \cdot d\vec{r}$ along the curve C which is the boundary of the right-angled triangle joining the points $(0,0)^t, (1,0)^t$ and $(0,2)^t$.
7. Compute $\int_C (e^x - y)dx + (\sin y + x) dx$. Here C is the boundary of the region consisting of points lying bounded by the curves $y = x^2$ and $y = 4$.
8. Sketch the gradient vector field of the function $f(x, y) = x^2 - y^2$ in the plane.