The Chinese University of Hong Kong

Department of Mathematics

MMAT5380 Graph Theory and Networks

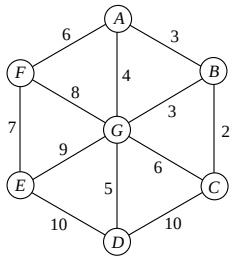
Assignment 4

Please hand in your assignment to the assignment box or the tutor before 6:40p.m. on Nov. 11, 2019 (Monday).

The assignment box is located at the 2nd floor of LSB and opposites to the Room 223.

4-1: Suppose G is bipartite. Show that if W is a closed walk of G, then the length of W is even. Note that, you are not allowed using Theorem 3.1.14 to prove this exercise since the proof of Theorem 3.1.14 is applied this exercise.

4-2: Consider the weighted graph

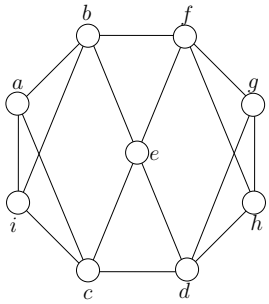


(a) Using Kruskal's algorithm find a minimal spanning tree. Show the list of chosen edges only.

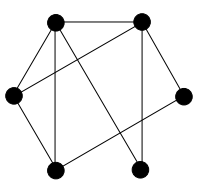
(b) Using Prim's algorithm in tabular form starting at D find a minimal spanning tree. Show your working steps.

4-3: (a) Apply DFS Spanning Tree Algorithm to the following graph G with vertex pre-ordering a, b, c, e, i, h, g, d, f.

(b) Using the Hopcroft and Tarjan algorithm, find a strongly orientation for the following graph.



4-4: Find the closure of the following graph G (step by step).



4-5: A mouse eats his way through a $3 \times 3 \times 3$ cube of cheese, tunneling through all 27 of the $1 \times 1 \times 1$ cubes. If the mouse starts at a corner, can it finish in the center?

4-6: Give an example of the following graph.

(a) Hamiltonian (simple) graph of order p but has at least a pair of vertices u and v satisfying $\deg(u) + \deg(v) < p$.

(b) Nonhamiltonian (simple) graph of order p but $\deg(v) \geq \frac{p-1}{2}$ for every vertex v.

4-7: Prove that if a simple connected graph G has an *independent set* X (i.e., a set of vertices in which no two vertices are adjacent) such that $|X| > |N(X)|$, then G is not Hamiltonian.

4-8: Prove that if $\deg(u) + \deg(v) \geq |G| - 1$ for any two nonadjacent vertices u, v of a simple connected graph G, then G contains a Hamiltonian path. [Hint: Use Corollary 4.2.9]