The Chinese University of Hong Kong

Department of Mathematics

MMAT5380 Graph Theory and Networks

Assignment 1

Please hand in your assignment to the assignment box or the tutor before 6:40p.m. on Sept. 30, 2019 (Monday).

The assignment box is located at the 2nd floor of LSB and opposites to the Room 223.

1-1: Draw a graph with the following vertex set and edge set.

(a) Vertex set: $\{u, v, w, x, y\}$ and edge set: $\{uv, vw, vx, wx, yv, wy\}$

(b) Vertex set: $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and edge set: $\{12, 23, 33, 34, 35, 67, 78, 18, 53, 71\}$.

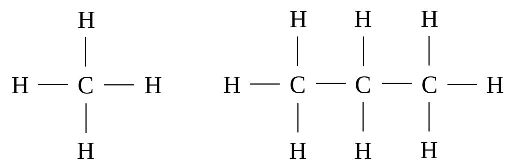
1-2: For the graph G defined in 1-1(b),

(a) List the degree sequence of G.

(b) Verify Handshaking Lemma for the graph G.

(c) Write down the adjacency matrix and the incidence matrix of G according to the vertex-list and edge-list above.

1-3: Figure represents the chemical molecules of methane (CH_4) and propane (C_3H_8).



(a) Regarding these diagrams as graphs, what can you say about the vertices representing carbon atoms (C) and hydrogen atoms (H)?

(b) There are two different chemical molecules with formula C_4H_{10}. Draw the graphs corresponding to these molecules.

1-4: (a) Draw a graph on 6 vertices with degree sequence $(5, 5, 5, 5, 3, 3)$; does there exist a simple graph with this degree sequence? Explain your answer.

(b) How are your answers to part (a) changed if the degree sequence is $(5, 5, 4, 3, 3, 2)$? Explain your answer.

1-5: (a) Determine which graphs in Figure 1 are subgraphs of the graph G. Explain your answer.

(b) Which graphs in Figure 1 are induced subgraphs of G. Explain your answer.

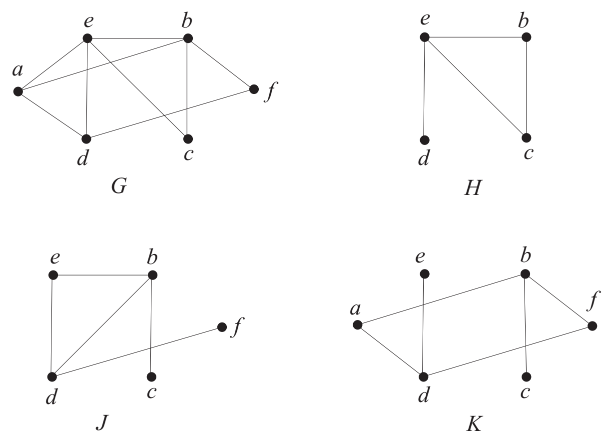
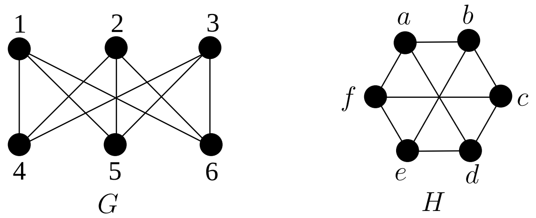


Figure 1:

1-6: Prove that the following two graphs are isomorphic by finding an appropriate isomorphism (show all the corresponding vertices and edges).



1-7: (a) Draw the Cartesian product $C_3 \times P_4$.

(b) Assume that G and H are graphs with $V(G) = \{u_1, u_2, \ldots, u_m\}$ and $V(H) = \{v_1, v_2, \ldots, v_n\}$, respectively. Let $(u_i,\, v_j)$ be a vertex in $G \times H$. Prove that $\deg_{G\times H}(u_i,\, v_j) = \deg_G(u_i) + \deg_H(v_j)$.

1-8: Suppose $G = (V, E)$ is a graph and A as well as B are subsets of V. Show the following statements:

(a) $N(A \cup B) = N(A) \cup N(B)$,

(b) $N(A \cap B) \subseteq N(A) \cap N(B)$.

END